A Social Network Analysis of Occupational Segregation*

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This version: October 2014

Abstract

We develop a network model of occupational segregation between social groups divided along gender or racial dimensions, generated by the existence of positive assortative matching among individuals from the same group. If referrals are important for job search, then expected homophily in the structure of job contact networks induces different career choices for individuals from different social groups. This further translates into stable occupational segregation equilibria in the labor market. We derive conditions for wage and unemployment inequality in the segregation equilibria and characterize both 1st and 2nd best social welfare optima. Surprisingly, we find that utilitarian socially optimal policies always involve segregation, but that additional distributional concerns can justify integration policies.

JEL codes: J24, J31, J70, Z13

Keywords: Social Networks, Homophily, Occupational Segregation, Labor Market Inequality, Social Welfare

*For useful discussion and suggestions at various stages of this paper, we are grateful to Willemien Kets, Michèle Belot, Sanjeev Goyal, Wilbert Grevers, Maarten Janssen, Joan de Martí, Friederike Mengel, James Montgomery, José Luis Moraga-González, Wojciech Olszewski, Gilles Saint-Paul, Ott Toomet, Jan van Ours, Fernando Vega-Redondo, and Yves Zenou. We also thank audiences from seminars at Babes-Bolyai University, Northwestern University, CPB Netherlands Bureau for Economic Policy Analysis, Aarhus University, Tartu University, Tinbergen Institute, University College London; and conferences: ESPE in Braga, SED in Montreal, GAMES in Evanston, CTN in Venice, EEA in Vienna, SMYE in Geneva, WEHIA in Essex, NAKE in Utrecht. Older working versions of this paper have acquired citations in a number of articles and books: we note that our main results and their interpretation from these earlier versions remain valid here. Buhai acknowledges financial support provided through a European Commission FP7 Marie Curie IOF (grant PIOF-GA-2009-255597). Van der Leij acknowledges financial support from the Spanish Ministry of Education and Science (project SEJ2007-62656). The usual disclaimers apply.

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1 Introduction

Occupational segregation between various social groups is an enduring and pervasive phenomenon, with important implications for the labor market. Richard Posner pointed out that "a glance of the composition of different occupations shows that in many of them, particularly racial, ethnic, and religious groups, along with one or the other sex and even groups defined by sexual orientation (heterosexual vs. homosexual), are disproportionately present or absent"[1]. There are countless empirical studies within sociology and economics that document the extent of occupational segregation. Most studies investigating the causes of occupational segregation agree that 'classical' theories such as taste or statistical discrimination by employers cannot alone explain occupational disparities and their remarkable persistence. While several meritorious alternative theories were to date considered, scientists with long-standing interest in the area, such as Kenneth Arrow (1998), particularly referred to modeling social network interactions as a promising avenue for further research in this context.

In this paper we consider a simple social interactions model in order to investigate a potential network channel leading to occupational segregation and wage inequality in the labor market. We construct a four-stage model of occupational segregation between two homogeneous, exogenously given, mutually exclusive social groups (e.g., genders, or races) acting in a two-job labor market. In the first stage each individual chooses one of two specialized educations to become a worker. In the second stage individuals randomly form "friendship" ties with other individuals, with a tendency to form relatively more ties with members of the same social group, what is known in the literature as "*(inbreeding) homophily*", "*inbreeding bias*" or "*assortative matching*".[2] In the third stage workers use their networks of friendship contacts to search for jobs. In the fourth stage workers earn a wage and spend their income on a single consumption good.

We obtain the following results. First, unsurprisingly, we show that with inbreeding homophily within social groups, a complete polarization in terms of occupations across the two

[1] The quote is from an essay entitled "Larry Summers and Women Scientists", posted at the "The Becker-Posner Blog", on 30-01-2005 (last accessed on 20 October 2014, at http://www.becker-posner-blog.com/2005/01/larry-summers-and-women-scientists--posner.html). Posner goes on by giving a clear-cut example of gender occupational segregation: "a much higher percentage of biologists than of physicists are women, and at least one branch of biology, primatology, appears to be dominated by female scientists. It seems unlikely that all sex-related differences in occupational choice are due to discrimination"

[2] Homophily measures the relative frequency of within-group versus between-group friendships. There exists inbreeding homophily or an inbreeding bias if the group's homophily is higher than what would have been expected if friendships are formed randomly. See Currarini, Jackson and Pin (2009) for formal definitions.

groups arises as a stable equilibrium outcome. This result follows from standard arguments on network effects. If a group is completely segregated and specialized in one type of job, then each individual in the group has many more job contacts if she "sticks" to her specialization. Hence, sticking to one specialization ensures good job opportunities to group members, and these incentives stabilize segregation.

We next extend the basic model allowing for “good” and “bad” jobs, in order to analyze equilibrium wage and unemployment inequality between the two social groups. We show that with large differences in job attraction (=wages), the main outcome of the model is that one social group "fully specializes" in the good job, while the other group "mixes" over the two jobs. In this partial segregation equilibrium, the group that specializes in the good job always has a higher payoff and a lower unemployment rate. Furthermore, with a sufficiently large intra-group homophily, the fully-specializing group also has a higher equilibrium employment rate *and* a higher wage rate than the "mixing" group, thus being twice advantaged. Hence, our model is able to explain typical empirical patterns of gender, race, or ethnic labor market inequality. The driving force behind our result is the fact that the group that fully specializes, being homogenous occupationally, is able to create a denser job contact network than the mixing group.

We finally consider whether society benefits from an integration policy, in the sense that labor inequality between the social groups would be attenuated. To this aim, we analyze a social planner’s first and second-best policy choices. Surprisingly, segregation is the preferred outcome in the first-best analysis, while a laissez-faire policy leading to segregation shaped by individual incentives is maximizing social welfare in the second-best case. Hence, overall employment is higher under segregation, while laissez-faire inequality remains sufficiently constrained, such that segregation is an overall socially optimal policy. We show that integration policies are justified only in the presence of additional distribution concerns, beyond individual utilities. Our social welfare analysis points out therefore some policy issues typically ignored in debates concerning anti-segregation legislature.

This paper is in several respects related to the segregation framework of Roland Benabou (1993).[3] Benabou introduces a model in which individuals choose between high and low education. The benefits of education, wages, are determined in the global labor market, but the costs are determined by local education externalities. In particular, the costs of high education are considerably more reduced than the costs of low education if many neighbors are highly

[3] The precursor of many studies on segregation is the seminal work by Schelling (1971), on the emergence of neighborhood racial segregation from tiny differences in the tolerance threshold levels of members of each of two races, regarding the presence of members of the other race.

educated as well, leading to underinvestment in education in the low-education neighborhoods. Benabou shows that these local education externalities lead to segregation and also to inequality at the macro level.[4] In our model, the link between local externalities and global outcomes is modeled similarly. However, there are a few essential modeling differences leading to markedly different implications. In Benabou (1993) agents choose *different* education levels, either high or low, and thus the marginal productivity and the wage are naturally higher for high-educated workers. Hence, in a segregation equilibrium the highly educated group (or neighborhood, in Benabou's model) has a natural wage advantage. This implies that, under the education externality mechanism, differences in education levels should fully explain the wage gap. As we discuss in more detail in Section 2.1, although there is evidence that ability and education differences might account to a considerable extent for the racial wage gap, such differentials cannot explain the gender wage gap. Moreover, in Benabou there is no involuntary unemployment and therefore the documented unemployment differences between races or genders remain untackled.

One of the key differences between this model and Benabou's concerns the results on social welfare. Whereas Benabou suggests that under education externalities *integration* may be the socially optimal policy, we argue here, in contrast, that a social planner would like to *segregate* society. The reason for this different outcome is that the education externalities flow only from high to low education in Benabou's framework—low educated agents "learn" from high educated agents—whereas externalities are symmetric in our model and thus equally beneficial to both groups. Intuitively, in Benabou (1993) segregation harms the group that has no high-educated agents and this group is better off by enforcing integration. On the other hand, in this paper contact networks are always more effective *for both groups,* when there is segregation. Our paper thus shows that a subtle difference in the mechanism of the local externalities can have major implications on optimal social policy.

Significant progress has been recently achieved in modeling labor market phenomena by means of social networks. Such articles have for instance investigated the effect of social networks on employment, wage inequality, and labor market transitions.[5] This work points out that individual performance on the labor market crucially depends on the position individuals take in the social network structure. However, these studies typically do not focus on the role that networks play in accounting for persistent patterns of occupational segregation and inequality

[4] Furthermore, by introducing the option to drop out of the labor market, Benabou shows that some neighborhoods may turn into ghettos of drop-outs, which has a dramatic impact on total welfare.

[5] The seminal paper on the role of networks in the labor markets is Montgomery (1991). Other well cited papers include, e.g., Calvó-Armengol and Jackson (2004, 2007), Ioannides and Soutevent (2006), Bramoullé and Saint-Paul (2010).

between races, genders or ethnicities.[6] Here, instead of focusing on the network structure, we take a simple reduced form approach, and we emphasize the mechanism relating the role of the job networks in the labor market to occupational segregation and inequality between social groups.

The paper is organized as follows. The next section shortly overviews empirical findings on occupational segregation. We review empirical evidence on the relevance of job contact networks and the extent of social group homophily in Section 2; we set up our model of occupational segregation in Section 3; and we discuss key results on the segregation equilibria in Section 4. Section 5 analyses the social welfare outcome. We summarize and conclude the paper in Section 6.

2 Empirical background

In this section we present the empirical background that motivates the building blocks of our model. We first discuss evidence on occupational segregation, and the relation to gender and race wage gaps. Next we overview some empirical literature on the role of job contact networks and on homophily.

2.1 The extent of occupational segregation

Although labor markets have become more open to traditionally disadvantaged groups, wage differentials by race and gender remain stubbornly persistent. Altonji and Blank (1999) give an overview of the literature on this topic. They note for instance that in 1995 a full-time employed white male earned on average \$ 42,742, whereas a full-time employed black male earned on average \$ 29,651, thus 30% less, and an employed white female \$ 27,583, that is, 35% less. Standard wage regressions are typically able to explain only half of this gap, but more detailed analysis reveals more insights. In particular, several authors have found that the inclusion of individual scores at the Armed Forces Qualifying Test is able to fill the wage gap on race, see the discussion in Altonji and Blank (1999) and the references therein. On the one hand, this suggests that the gap between whites and blacks is created before individuals enter the labor market. On the other hand, the gender wage gap cannot be fully accounted for by pre-market factors, as men and women usually have similar levels of education nowadays.

[6] Calvó-Armengol and Jackson (2004) find that two groups with two different networks may have different employment rates due to the endogenous decision to drop out of the labor market. However, their finding draws heavily on an example that already assumes a large amount of inequality; in particular, the groups are initially unconnected and the initial employment state of the two groups is unequal.

Much research within social sciences suggests that segregation into separate type of jobs, i.e. occupational segregation, explains a large part of the gender wage gap, as well as part of the race wage gap. A few examples of studies that review and/or present detailed statistics on the occupational segregation[7] and wage inequality patterns by gender, race or ethnicity are Beller (1982), Albelda (1986), King (1992), Padavic and Reskin (2002), Charles and Grusky (2004). All these studies agree that, despite substantial expansion in the labor market participation of women and affirmative action programs aimed at labor integration of racial and ethnic minorities, women typically remain clustered in female-dominated occupations, while blacks and several other races and ethnic groups are over-represented in some occupations and under-represented in others; these occupations are usually of lower 'quality', meaning they are paying less on average, which explains partly the male-female and white-black wage differentials[8].

King (1992) offers for instance detailed evidence that throughout 1940-1988 there was a persistent and remarkable level of occupational segregation by race and sex, such that "approximately two-thirds of men or women would have to change jobs to achieve complete gender integration", with some changes in time for some subgroups. Whereas occupational segregation between white and black women appears to have diminished during the 60's and the 70's, occupational segregation between white and black males or between males and females remained remarkable stable. Several studies by Barbara Reskin and her co-authors, c.f. the discussion and references in Padavic and Reskin (2002), document the extent of occupational segregation by narrow race-sex-ethnic cells and find that segregation by gender remained extremely prevalent and that within occupations segregated by gender, racial and ethnic groups are also aligned along stable segregation paths. Though most of these studies are for the USA, there is also international evidence (particularly from Europe) confirming that, with some variations, similar patterns of segregation hold, e.g. Pettit and Hook (2005).

[7] Some of these papers, e.g. Sørensen (2004), discuss in detail the extent of labor market segregation between social groups, at the *workplace, industry and occupation* levels. Here we shall be concerned with modeling segregation by *occupation* alone (known also as "horizontal segregation"), which appears to be dominant at least relative to segregation by industry. Weeden and Sørensen (2004) convincingly show that occupational segregation in the USA is much stronger than segregation by industries and that if one wishes to focus on one single dimension, "occupation is a good choice, at least relative to industry".

[8] The other prominent side of the 'labor market segregation explaining the wage penalty' story is that women relative to men and, respectively, blacks vis-à-vis whites might experience wage differentials within the same occupation, when located in different workplaces; then we deal with the so-called *vertical segregation* dimension. As stated above, we shall be concerned in this paper only with the occupational dimension, i.e. *horizontal segregation*.

2.2 Job contact networks

There is by now an established set of facts showing the importance of the informal job networks in matching job seekers to vacancies. For instance, on average about 50 percent of the workers obtain jobs through their personal contacts, e.g. Rees (1966), Granovetter (1995), Holzer (1987), Montgomery (1991), Topa (2001); Bewley (1999) enumerates several studies published before the 90's, where the fraction of jobs obtained via friends or relatives ranges between 30 and 60 percent[9]. It is also established that on average 40-50 percent of the employers actively use social networks of their current employees to fill their job openings, e.g. Holzer (1987). Furthermore, employer-employee matches obtained via contacts appear to have some common characteristics. Those who found jobs through personal contacts were on average more satisfied with their job, e.g. Granovetter (1995), and were less likely to quit, e.g. Datcher (1983), Devine and Kiefer (1991), Simon and Warner (1992), Datcher Loury (2006). For a more detailed overview of studies on job information networks, Ioannides and Datcher Loury (2004) is a recent reference.

2.3 Intra-group homophily

There is considerable evidence on the existence of the so-called social "homophily"[10], also labeled "assortative matching" or "inbreeding social bias", that is, there is a higher probability of establishing links among people with similar characteristics. Extensive research shows that people tend to be friends with similar others, see for instance McPherson et al. (2001) for a review, with characteristics such as race, ethnicity or gender being essential dimensions of homophily. It has also been documented that friendship patterns are more homophilous than would be expected by chance or availability constraints, even after controlling for the unequal distribution of races or sexes through social structure, e.g. Shrum, Cheek and Hunter (1988). There are also studies pointing towards "pure" same race preferences in marrying or dating (e.g. the "mating taboo" in Wong 2003 or the speed dating preferences in Fishman et al 2006), among very young kids (e.g. Hraba and Grant 1970) or among audiences of television shows (Dates 1980, Lee 2006).

[9] The difference in the use of informal job networks among professions is also documented. Granovetter (1995) pointed out that although personal ties seem to be relevant in job search-match for all professions, their incidence is higher for blue-collar workers (50 to 65 percent) than for white-collar categories such as accountants or typists (20 to 40 percent). However, for certain other white-collar categories, the use of social connection in job finding is even higher than for blue-collars, e.g. as high as 77 percent for academics.

[10] The "homophily theory" of friendship was first introduced and popularized by the sociologists Paul F. Lazarsfeld and Robert K. Merton (1954).

In our "job information network" context, early studies by Rees (1966) and Doeringer and Piore (1971) showed that workers who had been asked for references concerning new hires were in general very likely to refer people "similar" to themselves. While these similar features could be anything, such as ability, education, age, race and so on, the focus here is on groups stratified along exogenous characteristics (i.e. one is born in such a group and cannot alter her group membership) such as those divided along gender, race or ethnicity lines. Indeed, most subsequent evidence on homophily was in the context of such 'exogenously given' social groups. For instance, Marsden (1987) finds using the U.S. General Social Survey that personal contact networks tend to be highly segregated by race, while other studies such as Brass (1985) or Ibarra (1992), using cross-sectional single firm data, find significant gender segregation in personal networks. More recent evidence is also given by Mayer and Puller (2008) and Currarini et al. (2009).

Direct evidence of large gender homophily within job contact networks comes from tabulations in Montgomery (1992). Over all occupations in a US sample from the National Longitudinal Study of Youth, 87 percent of the jobs men obtained through contacts were based on information received from other men and 70 percent of the jobs obtained informally by women were as result of information from other women. Montgomery shows that these outcomes hold even when looking at each narrowly defined occupation categories or one-digit industries[11], including traditionally male or female dominated occupations, where job referrals for the minority group members were obtained still with a very strong assortative matching via their own gender group. For example, in male-dominated occupations such as machine operators, 81 percent of the women who found their job through a referral, had a female reference. Such figures are surprisingly large and are likely to be only lower bounds for magnitudes of inbreeding biases within other social groups[12].

Another relevant piece of evidence is the empirical study by Fernandez and Sosa (2005) who use a dataset documenting both the *recruitment* and the *hiring* stages for an entry-level job at a call center of a large US bank. This study also finds that contact networks contribute to the

[11] Weeden and Sørensen (2004) estimate a two-dimensional model of gender segregation, by industry *and* occupation: they find much stronger segregation across occupations than across industries. 86% of the total association in the data is explained by the segregation along the occupational dimension; this increases to about 93% once industry segregation is also accounted for. See also footnote 8.

[12] The gender homophily is likely to be smaller than race or ethnic homophily, given frequent close-knit relationships between men and women. This is confirmed for instance by Marsden (1988), who finds strong inbreeding biases in contacts between individuals of the same race or ethnicity, but less pronounced homophily within gender categories.

gender skewing of jobs, in addition documenting directly that there is strong evidence of gender homophily in the refereeing process: referees of both genders tend to strongly produce same sex referrals.

Finally, we briefly address the relative importance of homophily within "exogenously given" versus "endogenously created" social groups. As mentioned above, assortative matching takes place along a great variety of dimensions. However, there is empirical literature suggesting that homophily within exogenous groups such as those divided by race, ethnicity, gender, and- to a certain extent- religion, typically outweighs assortative matching within endogenously formed groups such as those stratified by educational, political or economic lines. E.g., Marsden (1988) finds for US strong inbreeding bias in contacts between individuals of the same race or ethnicity and less pronounced homophily by education level. Another study by Tampubolon (2005), using UK data, documents the dynamics of friendship as strongly affected by gender, marital status and age, but not by education, and only marginally by social class. These facts motivate why we focus here on "naturally" arising social groups, such as gender, racial or ethnic ones; nevertheless, as will become clear in the modeling, assuming assortative matching by education, *in addition* to gender, racial or ethnic homophily, does not matter for our conclusions.

3 A model of occupational segregation

Based on the stylized facts mentioned in Section 2.2, we build a parsimonious theoretical model of social network interaction able to explain stable occupational segregation, and employment and wage gaps, without a need for alternative theories.

Let us consider the following setup. A continuum of individuals with measure 1 is equally divided into two social groups, Reds (R) and Greens (G). The individuals are ex ante homogeneous apart from their social color. They can work in two occupations, A or B. Each occupation requires a corresponding thorough specialized education (career track), such that a worker cannot work in it unless she followed that education track. We assume that it is too costly for individuals to follow both educational tracks. Hence, individuals have to choose their education track before they enter the labor market.[13]

Consider now the following order of events:

[13] For example, graduating high school students may face the choice of pursuing a medical career or a career in technology. Both choices require several years of expensive specialized training, and this makes it unfeasible to follow both career tracks.

1. Individuals choose one education in order to specialize either in occupation A or in occupation B;

2. Individuals randomly establish "friendship" relationships, thus forming a network of contacts;

3. Individuals participate in the labor market. Individual i obtains a job with probability s^i.

4. Individuals produce a single good for their firms and earn a wage w^i. They obtain utility from consuming goods that they buy with their wage.

We proceed with an elaboration of these steps.

3.1 Education strategy and equilibrium concept

The choice of education in the first stage involves strategic behavior. Workers choose the education that maximizes their expected payoff given the choices of other workers, and we therefore look for a Nash equilibrium in this stage. This can be formalized as follows.

Denote by μ_R and μ_G the fractions of Reds and respectively Greens that choose education A. It follows that $1 - \mu_X$ of group $X \in \{R, G\}$ chooses education B. The payoffs will depend on these strategies: the payoff of a worker of group X that chooses education A is given by $\Pi_A^X(\mu_R, \mu_G)$, and mutatis mutandis, $\Pi_B^X(\mu_R, \mu_G)$. Define $\Delta\Pi^X \equiv \Pi_A^X - \Pi_B^X$. The functional form of the payoffs is made more specific later, in subsection 3.4.

In a Nash equilibrium each worker chooses the education that gives her the highest payoff, given the education choices of all other workers. Since workers of the same social group are homogenous, a Nash equilibrium implies that if some worker in a group chooses education A (B), then no other worker in the same group should prefer education B (A). This implies that a pair (μ_R, μ_G) is an *equilibrium* if and only if, for $X \in \{R, G\}$, the following hold:[14]

$$\Delta\Pi^X(\mu_R, \mu_G) \leq 0 \text{ if } \mu_X = 0 \tag{1}$$

$$\Delta\Pi^X(\mu_R, \mu_G) = 0 \text{ if } 0 < \mu_X < 1 \tag{2}$$

$$\Delta\Pi^X(\mu_R, \mu_G) \geq 0 \text{ if } \mu_X = 1. \tag{3}$$

[14] The question whether the equilibrium is in pure or mixed strategies is not relevant, because the player set is a measure of identical infinitesimal individuals (except for group membership). Our equilibrium could be interpreted as a Nash equilibrium in pure strategies; then μ_X is the measure of players in group X choosing pure strategy A. The equilibrium could also be interpreted as a symmetric Nash equilibrium in mixed strategies; in that case the common strategy of all players in group X is to play A with probability μ_X. A hybrid interpretation is also possible.

Table 1: The probability of a tie between two individuals, depending on the group membership and education choice.

		Education	
		same	different
Social group	same	$p + \kappa + \lambda$	$p + \lambda$
	different	$p + \kappa$	p

To strengthen the equilibrium concept, we restrict ourselves to *stable equilibria*. We use a simple stability concept based on a standard myopic adjustment process of strategies, which takes place before the education decision is made. That is, we think of the equilibrium as the outcome of an adjustment process. In this process, individuals repeatedly announce their preferred education choice, and more and more workers revise their education choice if it is profitable to do so, given the choice of the other workers.[15] Concretely, we consider stationary points of a dynamic system guided by the differential equation $\dot{\mu}_X = k\Delta\Pi^X(\mu_R, \mu_G)$. This implies that $\mu \equiv (\mu_R, \mu_G)$ is a stable equilibrium if it is an equilibrium and (i) for $X \in \{R, G\}$: $\partial\Delta\Pi^X/\partial\mu_X < 0$ if $\Delta\Pi^X = 0$; (ii) $\det(D\Delta\Pi(\mu)) > 0$ if $\Delta\Pi^R = 0$ and $\Delta\Pi^G = 0$, where $D\Delta\Pi(\mu)$ is the Jacobian of $(\Delta\Pi^R, \Delta\Pi^G)$ with respect to μ.

3.2 Network formation

In the second stage the workers form a network of contacts. We assume this network to be random, but with social color homophily. That is, we assume that the probability for two workers to create a tie is $p \geq 0$ when the two workers are from different social groups and follow different education tracks; however, when the two workers are from the same social group, the probability of creating a tie increases with $\lambda > 0$. Similarly, if two workers choose the same education, then the probability of creating a tie increases with $\kappa \geq 0$. Hence, we allow for assortative matching by education, in addition to the one by social color. We do not impose any further restrictions on these parameters, other than securing $p + \lambda + \kappa \leq 1$. This leads to the tie formation probabilities from Table 1. We shall refer to two workers that create a tie as "friends"

[15] One could think of such a process as the discussions students have before the end of the high school about their preferred career. An alternative with a longer horizon is an overlapping generations model, in which the education choice of each new generation partly depends on the choice of the previous generation.

We assume the probability that an individual i forms a tie with individual j to be exogenously given and constant. In practice, establishing a friendship between two individuals typically involves rational decision making. It is therefore plausible that individuals try to optimize their job contact network in order to maximize their chances on the labor market.[16] In particular, individuals from the disadvantaged social groups should have an incentive to form ties with individuals from the advantaged group. While this argument is probably true, we do not incorporate this aspect of network formation in our model. The harsh reality is that strategic network formation does not appear to dampen the inbreeding bias in social networks significantly; in Section 2.2 we provided an abundance of evidence that strong homophily exists even within groups that have strong labor market incentives *not* to preserve such homophily in forming their ties. The reason could be that the payoff of forming a tie is mainly determined by various social and cultural factors, and only for a smaller part by benefits from the potential transmission of valuable job information.[17] On top of that, studies such as, for instance, Granovetter (2002), also note that many people would feel exploited if they find out that someone befriends them for the selfish reason of obtaining job information. These elements might hinder the role of labor market incentives when forming ties. Hence, while we do not doubt that incentives do play a role when forming ties, we believe these incentives are not sufficient to undo the effects of the social color homophily. We therefore assume network formation exogenous in this paper.

3.3 Job matching and social networks

The third stage we envision for this model is that of a dynamic labor process, in which information on vacancies is propagated through the social network, as in, e.g., Calvó-Armengol and Jackson (2004), Calvó-Armengol and Zenou (2005), Ioannides and Soetevent (2006) or Bramoullé and Saint-Paul (2006). Workers who randomly lose their job are initially unemployed because it takes time to find information on new jobs. The unemployed worker receives such information either directly, through formal search, or indirectly, through employed friends who receive the information and pass it on to her (in the particular case where all her friends are unemployed, only the formal search method works). As the specific details of such a process are not important for our purposes, we do not consider these dynamic models explicitly, but take a "reduced form" approach.

In particular, we assume that unemployed workers have a higher propensity to receive job information when they have more friends with the *same job background*, that is, with the *same*

[16] See Calvó-Armengol (2004) for a model of strategic network formation in the labor market.

[17] Currarini et al.(2009) discuss a model of network formation in which individuals form preferences on the number and mix of same-group and other-group friends. In this model inbreeding homophily arises endogenously.

choice of education. On the one hand, this assumption is based on the result of Ioannides and Soetevent (2006) that in a random network setting the individuals with more friends have a lower unemployment rate.[18] On the other hand, this assumption is based on the conjecture that workers are more likely to receive information about jobs in their own occupation. For example, when a vacancy is opened in a team, the other team members are the first to know this information, and are also the ones that have the highest incentives to spread this information around.

Formally, denote the probability that individual i becomes employed by $s^i = s(x_i)$, where x_i is the measure of friends of i with the same education as i has. We thus assume that $s(x)$ is differentiable, $0 < s(0) < 1$ (there is non-zero amount of direct job search) and $s'(x) > 0$ for all $x > 0$ (the probability of being employed increases in the number of friends with the same education).

It is instructive to show how s^i depends on the education choices of i and the choices of all other workers. Remember that μ_R and μ_G are the fractions of Reds and respectively Greens that choose education A. Given the tie formation probabilities from Table 1 and some algebra, the employment rate s_A^X of A-workers in group $X \in \{R, G\}$ will be given by:

$$s_A^X(\mu_R, \mu_G) = s\left((p+\kappa)\bar{\mu} + \lambda\mu_X/2\right) \tag{4}$$

and likewise, the employment rate s_B^X of B-workers in group X will be

$$s_B^X(\mu_R, \mu_G) = s\left((p+\kappa)(1-\bar{\mu}) + \lambda(1-\mu_X)/2\right) \tag{5}$$

where $\bar{\mu} \equiv (\mu_R + \mu_G)/2$.

Note that $s_A^X > s_A^Y$ and $s_B^X < s_B^Y$ for $X, Y \in \{R, G\}$, $X \neq Y$, if and only if $\mu_X > \mu_Y$ and $\lambda > 0$. We will see in Section 4.1 that the ranking of the employment rates is crucial, as it creates a group-specific network effect. That is, keeping this ordering, if only employment matters (jobs are equally attractive), then individuals have an incentive to choose the same education as other individuals in *their* social group. Importantly, it is straightforward to see that this ordering of the employment rates depends on λ, but it does not depend on κ. Therefore, only the homophily among members of the same social group- and not the eventual assortative matching by education- is relevant to our results.

[18] This result is nontrivial, as the unemployed friends of employed individuals tend to compete with each other for job information. Thus, if a friend of a jobseeker has more friends, the probability that this friend passes information to the jobseeker decreases. In fact, in a setting in which everyone has the same number of friends, Calvó-Armengol and Zenou (2005) show that the unemployment rate is non-monotonic in the (common) number of friends.

3.4 Wages, consumption and payoffs

The eventual payoff of the workers depends on the wage they receive, the goods they buy with that wage, and the utility they derive from consumption. Without loss of generality we assume that an unemployed worker receives zero wage. However, the wages of employed workers are not exogenously given, but they are determined by supply and demand.

When firms offer wages, they take into account that there are labor market frictions and that it is impossible to employ all workers simultaneously. Thus what matters is the effective supply of labor as determined by the labor market process in stage 3. Let L_A be the total measure of employed A-workers and L_B be the total measure of employed B-workers. Hence,

$$L_A(\mu_R, \mu_G) = \mu_R s_A^R(\mu_R, \mu_G)/2 + \mu_G s_A^G(\mu_R, \mu_G)/2 \tag{6}$$

and

$$L_B(\mu_R, \mu_G) = (1 - \mu_R) s_B^R(\mu_R, \mu_G)/2 + (1 - \mu_G) s_B^G(\mu_R, \mu_G)/2. \tag{7}$$

Given (4) and (5) from above, it is easy to check that L_A is increasing with μ_R and μ_G, whereas L_B is decreasing with μ_R, μ_G.

As in Benabou (1993), consumption, prices, utility, the demand for labor and the implied wages are determined in a 1-good, 2-factor general equilibrium model. All individuals have the same utility function $U : R_+ \to R$, which is strictly increasing and strictly concave with $U(0) = 0$. The single consumer good sells at unit price, such that consumption of this good equals wage and indirect utility is given by $U_i = U(w_i)$.

Firms put A-workers and B-workers together to produce the single good at constant returns to scale. Wages are then determined by the production function $F(L_A, L_B)$. As usually, we assume that F is strictly increasing and strictly concave in L_A and L_B and $\partial^2 F/\partial L_A \partial L_B > 0$. Writing the wage as function of education choices and using (6) and (7), the wages of A-workers and B-workers, w_A and w_B, are given by

$$w_A(\mu_R, \mu_G) = \frac{\partial F}{\partial L_A}\left(L_A(\mu_R, \mu_G), L_B(\mu_R, \mu_G)\right),$$

and

$$w_B(\mu_R, \mu_G) = \frac{\partial F}{\partial L_B}\left(L_A(\mu_R, \mu_G), L_B(\mu_R, \mu_G)\right).$$

It is easy to check that w_A is strictly decreasing with μ_R and μ_G, and mutatis mutandis, w_B.

We can now define the payoff of a worker as her expected utility at the time of decision-making. The payoff function of an A-educated worker from social group $X \in \{R, G\}$ is thus

$$\Pi_A^X(\mu_R, \mu_G) = s_A^X(\mu_R, \mu_G) U(w_A(\mu_R, \mu_G)). \tag{8}$$

Similarly,

$$\Pi_B^X(\mu_R, \mu_G) = s_B^X(\mu_R, \mu_G)U(w_B(\mu_R, \mu_G)). \tag{9}$$

If we do not impose further restrictions, then there could be multiple equilibria, most of them uninteresting. To ensure a unique equilibrium in our model (actually: two symmetric equilibria), we make the following two assumptions.

Assumption 1 *For the wage functions w_A and w_B*

$$\lim_{x \downarrow 0} U(w_A(x, x)) = \lim_{x \downarrow 0} U(w_B(1-x, 1-x)) = \infty.$$

Assumption 2 *For $X \in \{R, G\}$, and for all $\mu_R, \mu_G \in [0, 1]$*

$$\left| \frac{\partial s_A^X / s_A^X}{\partial \mu_X / \mu_X} \right| < \left| \frac{\partial U/U}{\partial w_A / w_A} \right| \left| \frac{\partial w_A / w_A}{\partial \mu_X / \mu_X} \right|$$

and

$$\left| \frac{\partial s_B^X / s_B^X}{\partial \mu_X / \mu_X} \right| < \left| \frac{\partial U/U}{\partial w_B / w_B} \right| \left| \frac{\partial w_B / w_B}{\partial \mu_X / \mu_X} \right|.$$

Assumptions 1 and 2 guarantee the uniqueness of our results. Assumption 1 implies that the wage for scarce labor is so high that at least some workers always find it attractive to choose education A or respectively B; everyone going for one of the two educations cannot be an equilibrium. In Assumption 2 we assume that the education choice of an individual has a smaller marginal effect on the employment probability within a group than on the wages and overall utility. Note that the assumption implies that for $X \in \{R, G\}$

$$\frac{\partial \Pi_A^X}{\partial \mu_X} < 0 < \frac{\partial \Pi_B^X}{\partial \mu_X},$$

and it is this feature that guarantees the uniqueness of our results. The assumption is not restrictive as long as there is sufficient direct job search, because the employment probability of each individual in our model is bounded between $s(0) > 0$ and 1, with $s(0)$ capturing the employment probability in the absence of any ties and thus induced only by the exogenously given direct job finding rate. Hence, a higher $s(0)$ implies less of an impact of the network effect on the employment rate.

It should be noted that we make these assumptions above only in order to focus our analysis on segregation outcomes, for the sake of clarity and brevity. These assumptions are not necessary. For instance, in the calibration exercise of Section 5.2.1, Assumption 2 is violated, but there are still (two) unique equilibria.

4 Equilibrium results

We now present the equilibrium analysis of our model. The formal proofs of all subsequent propositions are relegated to the Appendix. Without loss of generality we assume throughout the section that $w_A(1,0) \geq w_B(1,0)$, thus that the A-occupation is weakly more attractive than the B-occupation when effective labor supply is equal. We call A the "good" job, and B the "bad" job.

4.1 Occupational segregation

We are in particular interested in those equilibria in which there is segregation. We define *complete segregation* if $\mu_R = 0$ and $\mu_G = 1$, or, vice versa, $\mu_R = 1$ and $\mu_G = 0$. On the other hand, we say that there is *partial segregation* if for $X \in \{R, G\}$ and $Y \in \{R, G\}$, $Y \neq X$: $\mu_X = 0$ but $\mu_Y < 1$, or, vice versa, $\mu_X = 1$ but $\mu_Y > 0$.

Our first result is that segregation, either complete or partial, is the only stable outcome:

Proposition 1 *Suppose Assumptions 1 and 2 hold. Define* $s_H \equiv s((p+\kappa+\lambda)/2)$ *and* $s_L \equiv s((p+\kappa)/2)$.

(i) If

$$1 \leq \frac{U(w_A(1,0))}{U(w_B(1,0))} \leq \frac{s_H}{s_L}, \tag{10}$$

then there are exactly two stable equilibria, both with complete segregation.

(ii) If

$$\frac{U(w_A(1,0))}{U(w_B(1,0))} > \frac{s_H}{s_L}, \tag{11}$$

then there are exactly two stable equilibria, both with partial segregation, in which either $\mu_R = 1$ *or* $\mu_G = 1$.

We first note that a non-segregation equilibrium cannot exist, even in the case of a tiny amount of homophily λ. The intuition is that homophily in the social network among members of the same social group creates a group-dependent network effect. Thus, if slightly more Red workers choose A than Greens do, then the value of an A-education is higher for the Reds than for the Greens, while the value of a B-education is lower in the Reds' group. Positive feedback then ensures that the initially small differences in education choices between the two groups widen and widen, until at least one group segregates completely into one type of education.

Second, if the wage differential between the two jobs (for equal numbers of A-educated and B-educated workers) is not "too large" vis-à-vis the social network effect (condition 10), complete segregation is the only stable equilibrium outcome, given a positive inbreeding bias in the social group. Thus one social group specializes in one occupation, and the other group in the other occupation. On the other hand, the proposition makes clear that complete segregation cannot be sustained if the wage differential is "too large" vis-à-vis the social network effect (condition 11). Starting from complete segregation, a large wage differential gives incentives to the group specialized in B-jobs to switch to A-jobs.

Interestingly, the "unsustainable" complete segregation equilibrium is then replaced by a partial equilibrium in which one group specializes in the "good" job A, while the other group has both A and B-workers. Partial segregation in which one group, say the Greens, fully specializes in the "bad" job B is unsustainable, as that would lead to an oversupply of B-workers and an even larger wage differential. This would provide the Red B-workers with strong incentives to switch en masse to the A-occupation.

4.2 Inequality

The discussion so far ignored eventual equilibrium differentials in wages and unemployment between the two types of jobs. We now tackle that case. We continue to assume that $w_A(1,0) \geq w_B(1,0)$ and, in light of the results of Proposition 1, we focus without loss of generality on the equilibrium in which $\mu_R = 1$. Thus, the Reds specialize in the "good" job A, while the "bad" job B is only performed by Green workers.

We first consider the case in which wage differentials are small enough so that complete segregation is an equilibrium ($\mu_R = 1$ and $\mu_G = 0$). In this case the implications are straightforward. Since both groups specialize in equal amounts, the network effects are equally strong, and the employment rates are equal. Given that employment rates are equal, the effective labor supply is also equal, and therefore the wage of the "good" job is weakly higher. We thus have the following result:

Proposition 2 *Suppose Assumptions 1 and 2 hold. Define* $s_H \equiv s((p+\kappa+\lambda)/2)$ *and* $s_L \equiv s((p+\kappa)/2)$ *and suppose that* $1 \leq \frac{w_A(1,0)}{w_B(1,0)} \leq \frac{s_H}{s_L}$*. Suppose* $(\mu_R, \mu_G) = (1,0)$ *is a stable equilibrium. In that equilibrium*

$$w_A \geq w_B,$$

$$s_A^R = s_B^G > s_B^R = s_A^G,$$

and

$$\Pi_A^R \geq \Pi_B^G \geq \Pi_A^G \geq \Pi_B^R. \tag{12}$$

This result is not very surprising, hence we turn next to the analysis of the more interesting case in which wage differentials are large. In that case there is a partial equilibrium in which $(\mu_R, \mu_G) = (1, \mu^*)$ where $\mu^* \in (0, 1)$. First note that according to (2) this implies the following condition:

$$\Pi_A^G(1, \mu^*) = \Pi_B^G(1, \mu^*),$$

or equivalently

$$s_A^G(1, \mu^*)U(w_A(1, \mu^*)) = s_B^G(1, \mu^*)U(w_B(1, \mu^*)).$$

Thus, whereas workers in group R prefer the A-job, the workers in group G make an individual trade-off: lower wages should be exactly compensated by higher employment probabilities and vice versa.

We are particularly interested in whether this individual trade-off between unemployment and wages translates into a similar trade-off at the 'macro-level', in which an inter-group wage gap is compensated by a reversed employment gap. We have the following proposition.

Proposition 3 *Suppose Assumptions 1 and 2 hold. Define $s_H \equiv s((p + \kappa + \lambda)/2)$ and $s_L \equiv s((p + \kappa)/2)$ and suppose that $\frac{w_A(1,0)}{w_B(1,0)} > \frac{s_H}{s_L}$. Define $\hat{\mu} \in (0, 1)$, such that*

$$w_A(1, \hat{\mu}) = w_B(1, \hat{\mu}), \tag{13}$$

and let $(\mu_R, \mu_G) = (1, \mu^*)$ be a stable equilibrium. In that equilibrium

$$\Pi_A^X > \Pi_B^Y = \Pi_A^Y > \Pi_B^X. \tag{14}$$

Moreover,

(i) if $\hat{\mu} < \frac{\lambda}{2(p+\kappa+\lambda)}$, then

$$s_A^R > s_B^G > s_A^G > s_B^R,$$

and

$$w_A(1, \mu^*) > w_B(1, \mu^*);$$

(ii) if $\hat{\mu} > \frac{\lambda}{2(p+\kappa+\lambda)}$*, then*

$$s_A^R > s_A^G > s_B^G > s_B^R,$$

and

$$w_B(1, \mu^*) > w_A(1, \mu^*).$$

The main implication of this proposition is that an inter-group wage gap is *not* compensated by a reversed employment gap. On the contrary, it is possible that the group specializing in the good job, here the Reds, both earns a higher wage *and* has higher employment probabilities than the Greens group. This is especially clear when the group homophily bias λ is large relative to p and κ (in fact $p + \kappa$) and there is a big difference in attractiveness between the good and the bad jobs (case (i) above).

This result can be understood by the following observation: the workers in the 'specializing' group R have a higher employment probability than *all* workers in group G. This is always the case, regardless of whether the individual in G is an A or a B worker, and whether $s_B^G > s_A^G$ or not. As all members of group R choose the same occupation, the Reds remain a strong homogenous social group. Network formation with homophily then implies that they are able to create a lot of ties, and hence, that they benefit most from their social network. On the other hand, the Greens are dispersed between two occupations. This weakens their social network and this decreases their chances on the labor market, both for A and B-workers in group G.

Whether the wage differential between the workers in the two groups is positive or negative depends on the relative size of λ relative to $p + \kappa$, in the term $\frac{\lambda}{2(p+\kappa+\lambda)}$ from the inequality conditions in Proposition 3. This can be roughly assessed in light of the empirical evidence on homophily discussed earlier in this paper. First, as seen from the stylized facts from Section 2.2, the assortative matching by education, κ, is typically found to be lower relative to racial, ethnical or gender homophily. The second interesting situation is a scenario where the probability of making contacts in general, p, were already extremely high relative to the intra-group homophily bias. However, given the surprisingly large size of intra-group inbreeding biases in personal networks of contacts found empirically, this is also unlikely. Hence, the likelihood is very high that in practice λ would dominate the other parameters in the cutoff term $\frac{\lambda}{2(p+\kappa+\lambda)}$.

Let us sum up the implications of this last proposition. The fully specializing group is always better off in terms of unemployment rate and payoff, independent of either relative or absolute sizes of λ, p and κ (as long as $\lambda > 0$), as shown in Proposition 3. Furthermore, given the observed patterns of social networks discussed in Section 2.2, the condition of λ dominant relative to p and κ is likely to be met. This ensures that the group fully specializing in the good job always has a higher wage in the equilibrium than the group mixing over the two jobs, as proved in Proposition 3. Note that this partial segregation equilibrium is in remarkable agreement with observed occupational, wage and unemployment disparities in the labor market between, for instance, males-females or blacks-whites. This suggests that our simple model offers a plausible explanation for major empirical patterns of labor market inequality.

5 Social welfare

5.1 First best social optimum

In the previous section we observed that individual incentives lead to occupational segregation and wage and unemployment inequality. This suggests that a policy targeting integration may reduce inequality as well, and in fact may just be socially beneficial. This is an argument often used for instance by proponents of positive discrimination. We set out here to analyze the implications of our model from a social planner's point of view.

Consider a utilitarian social welfare function:

$$W(\mu_R, \mu_G) = \mu_R \Pi_A^R/2 + (1-\mu_R)\Pi_B^R/2 + \mu_G \Pi_A^G/2 + (1-\mu_G)\Pi_B^G/2, \tag{15}$$

where $\Pi_A^X \equiv \Pi_A^X(\mu_R, \mu_G)$ and $\Pi_B^X \equiv \Pi_B^X(\mu_R, \mu_G)$ are given by equations (8) and (9). Since unemployed workers obtain zero utility, we can also write the welfare function as

$$W(\mu_R, \mu_G) = L_A U\left(\frac{\partial F}{\partial L_A}(L_A, L_B)\right) + L_B U\left(\frac{\partial F}{\partial L_B}(L_A, L_B)\right), \tag{16}$$

where $L_A \equiv L_A(\mu_R, \mu_G)$ and $L_B \equiv L_B(\mu_R, \mu_G)$ were introduced by (6) and (7). The formulation in (16) is useful, because it shows that what matters for social welfare is the effect of a policy on the society's effective labor supply.

We consider a first-best social optimum, that is, the social planner is able to fully manage $\mu_R \in [0,1]$ and $\mu_G \in [0,1]$ and therefore the social optimum $\mu^S = (\mu_R^S, \mu_G^S)$ is defined as

$$\mu^S = \text{argmax}_{\mu_R \in [0,1], \mu_G \in [0,1]} W(\mu_R, \mu_G).$$

We obtain the following result:

Proposition 4 *If for all $x \in [0, (p+\kappa+\lambda)/2]$:*

$$s''(x) > -\frac{4}{\lambda}\; s'(x) \tag{17}$$

then any social optima involves complete or partial segregation.

Thus a segregation policy is socially preferred, as long as $s(x)$, the employment probability of having x friends with the same education, is "not too concave". This proposition can be intuitively understood as follows. Suppose that there is no segregation, and $0 < \mu_G < \mu_R < 1$. In that case the Reds obtain a higher employment probability in an A-occupation, $s_A^R > s_B^R$, whereas the Greens have a higher employment rate as B-workers, $s_B^G > s_A^G$. Now consider the effect on segregation, wages and employment when a social planner forces a Red individual initially choosing a B-occupation and respectively, a Green individual initially choosing an A-occupation, into *switching their occupation choice*. In that case μ_R slightly increases, whereas μ_G slightly decreases. The result of this event is, first, that segregation increases; the gap between μ_R and μ_G becomes larger. Second, the total fraction of individuals that choose occupation A, $\mu_R + \mu_G$, does not change. So the ratio of A-workers versus B-workers does not change much, and therefore the ratio of wages does not change much either. Thus the effect on wage inequality is only marginal. Third, by switching occupations, the Red worker can now benefit from a denser network, and have an employment probability s_A^R instead of s_B^R. The same is true for the Green worker switching from B to A. Thus, the combined payoff of the two workers increases, as they are both more likely to become employed. We also need to consider the externality on the employment rates of the workers not involved in the occupation switch. In particular, the switch of occupations increases the network effects of the other Red A-workers and Green B-workers, whereas it decreases the network effects of Red B-workers and Green A-workers. The restriction on the concavity of $s(x)$ ensures that the switch of occupations puts on average a positive externality on the employment probabilities of other workers. We conclude that the switch of occupations of the two workers hardly affects wage inequality, while it increases the labor supply of both A and B. Therefore, social welfare increases.

The general message of this result is that an integration policy might have detrimental effects on employment, effects that are usually overlooked by strong advocates of positive discrimination. Under our model's assumptions, integration might weaken the employment chances of individuals, because the network effects are weaker in mixed networks. In the case of complete segregation, individuals are surrounded by similar individuals during their education. Therefore, it is easier for them to make many friends they can rely on when searching on the job market. Consequently, employment probabilities are high. On the other hand, if educations are mixed, then individuals have more difficulties in creating useful job contacts, and therefore their employment probabilities are lower.

It is worth to point out that the result that integration weakens network effects and decreases labor market opportunities has empirical support in related literature on segregation. For example, Currarini et al. (2009) find clear evidence that larger (racial) minorities create more friendships, and Marsden (1987) finds a similar pattern in his network of advice. Therefore, it is more beneficial for a worker to choose an education in which she is only surrounded by similar others, instead of an education in which racial groups are mixed, let alone one in which she is a small minority. In a different but related context, Alesina and La Ferrara (2000, 2002) find that participation in social activities is lower in racially mixed communities and so is the level of trust. These and our results suggest that possible negative impacts of integration on social network effects should also be taken into account.

Our outcome on the first-best social optimum hinges for a large part on the fact that the social planner is able to increase employment by increasing segregation, while still controlling wage inequality. In reality however, a social planner may not have this amount of control. Perhaps a more feasible policy is a policy in which the social planner enforces and stabilizes integration, but where the exact allocation of workers to occupations is determined by individual incentives. In the case of segregation there would be a potentially large inequality in payoffs between the social groups, whereas in the case of integration there may be complete payoff equality, but employment may be lower. This suggests a second-best analysis of social welfare, in which there is a potential trade-off of segregation between network benefits and inequality. Such an analysis is unfeasible without further specification of the parameters, hence we will perform that analysis subsequent to calibrating the model for suitable parameters and functional forms.

5.2 Second best social optimum

5.2.1 Numerical simulation

As often done in such frameworks, we calibrate the parameters, in order to perform a small numerical simulation of our model. The purpose of this simulation is to get a better feeling on the mechanisms of the model, the restrictiveness of our assumptions, and the magnitude of the wage gap that can be generated. The simulation also allows us to get some insights about a second-best welfare policy.

We first specify functional forms for $s(x)$, the employment probability as function of the number of friends with the same education, $F(L_A, L_B)$, the production function and thus the derived wage functions, and $U(x)$, the utility function. Regarding the employment probability, we consider a function that follows from a dynamic labor process, in which employed individuals become unemployed at rate 1, and in which unemployed individuals become employed at rate

$c_0 + c_1 x$, where c_0 is the rate at which unemployed workers directly obtain information on job vacancies, and c_1 measures the strength of having friends. This leads to the following employment function:

$$s(x) = \frac{c_0 + c_1 x}{1 + c_0 + c_1 x}.$$

Since we have defined $s_0 = s(0)$ as the employment probability when only direct search is used, it follows that $s_0 = c_0/(1 + c_0)$.

For the production function we assume the commonly used Cobb-Douglas function with constant returns to scale,

$$F(L_A, L_B) = \theta L_A^{\alpha} L_B^{1-\alpha}.$$

For the utility function we consider a function with constant absolute risk aversion, where ρ is the coefficient of absolute risk aversion. That is

$$U(x) = 1 - e^{-\rho x}.$$

We calibrate the parameters $s_0, c_1(p+\kappa), c_1\lambda, p$ and θ, leaving α as a free parameter. First, we calibrate s_0, $c_1(p+\kappa)$, and $c_1\lambda$ from three equations that are motivated by the empirical evidence given in Section 2 and 3. This parameterization is sufficient to perform the simulation, and it is thus not necessary to separately specify c_1, p, κ and λ. The first equation is obtained by imposing the restriction that about 50% of the workers find their job through friends, as suggested in Section 2. This restriction implies that the direct job arrival rate c_0 should equal the indirect job arrival rate through friends $c_1 x$. The indirect job arrival rate differs, depending on the choices of the individuals, but if we focus on the case complete segregation, in which $\mu_R = 1$ and $\mu_G = 0$, then we can impose the following restriction:

$$c_0 = c_1(p + \kappa + \lambda)/2.$$

Next, we calibrate the amount of inbreeding homophily in the social group. This amount typically differs depending on the group defining characteristic. For example, analyzing data on Facebook participants at Texas A&M, Mayer and Puller (2008) find that two students living in the same dorm are 13 more likely to be friends than two random students, two black students 17 more likely, but two Asian students 5 times more likely and two Hispanic students twice as likely to be friends. In light of this evidence, we chose to keep the amount of inbreeding homophily in the simulation modest, imposing $\lambda = 3(p + \kappa)$.

We next impose that the employment rate is 95% in case of complete segregation. Given the above, we solve

$$\frac{2c_0}{1 + 2c_0} = 0.95,$$

Table 2: Chosen parameter values in the simulation and the sensitivity with respect to $\hat{\alpha}$ and the maximum wage gap.

Parameter	Value	Elasticity of $\hat{\alpha}$ $\hat{\alpha} = .5904$	Elasticity of wage gap $G(1,0) = .306$
s_0	.9048	-1.71	-9.47
$c_1(p+\kappa)$	4.75	-.04	-.23
$c_1\lambda$	14.25	.08	.46
ρ	1.0×10^{-4}	.38	2.09
θ	80,000	.38	2.09

and this implies that

$$s_0 = \frac{c_0}{1+c_0} = \frac{19}{21} \approx .9048.$$

and further that $c_1(p+\kappa) = 4.75$ and $c_1\lambda = 14.25$.

Let us consider now the productivity parameter θ and the coefficient of absolute risk aversion ρ. The coefficient of absolute risk aversion has been estimated between 6.6×10^{-5} and 3.1×10^{-4} (Gertner 1993, Metrick 1995, Cohen and Einav 2007). We set the risk aversion at 1.0×10^{-4}, which means a coefficient of relative risk aversion of 4 at a wealth level of \$ 40,000, or indifference at participating in a lottery of getting \$ 100.00 or losing \$ 99.01 with equal probability.

The productivity parameter, θ, is chosen such that average income equal \$ 40,000 in the case of complete segregation, $(\mu_R, \mu_G) = (1, 0)$, and $\alpha = .5$.[19] Since in that situation $w_A(1,0) = w_B(1,0) = \theta/2$, we have $\theta = 80,000$.

We can now look at the dependence of payoffs, wages and employment on α with s_0, $c_1(p+\kappa)$, $c_1\lambda$, ρ and θ as summarized in Table 2, and in which μ_R and μ_G are determined by equilibrium conditions (1)-(3). Given the result of Proposition 1 that there is either a complete equilibrium or a partial equilibrium, in which one group specializes in the good job, we concentrate our attention to the parameter space in which $\alpha \in [1/2, 1)$, $\mu_R = 1$ and $\mu_G \in [0, 1)$. Thus occupation A is "good", and group R specializes in A.

We first show a plot of $\Delta\Pi^G(1, \mu_G)$ as a function of μ_G for different values of α. This function illustrates the payoff evaluation that a Green individual makes when deciding on its occupation. If $\Delta\Pi^G(1, \mu_G) > (<)0$, then the Green individual prefers A (B) if she beliefs that all Reds choose A and fraction μ_G of Greens choose A. Clearly, in an equilibrium it should hold that either $\Delta\Pi^G(1, 0) < 0$ or $\Delta\Pi^G(1, \mu_G) = 0$.

[19] GDP per capita was \$ 44,190 in the U.S. in 2006 according to figures from the IMF.

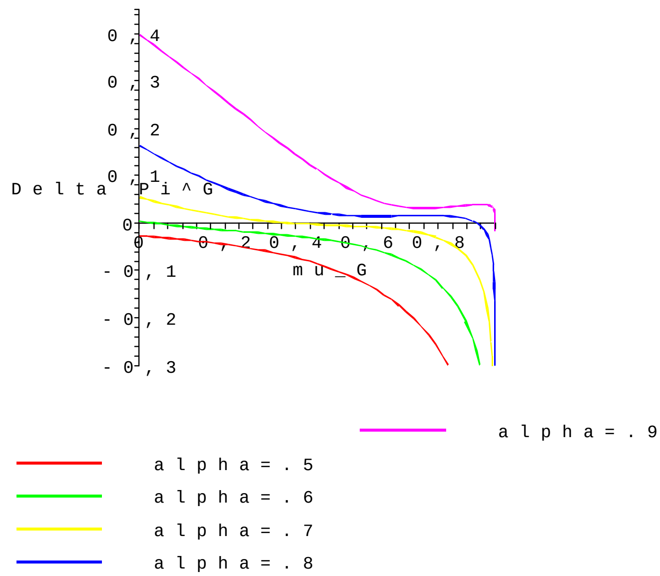

Figure 1: $\Delta\Pi^G(1,\mu_G)$ as a function of μ_G for different values of α.

The plot is displayed in Figure 1. This plot nicely illustrates the workings of the model. First, note that for $\alpha = .5$, $\Delta\Pi^G(1,\mu_G)$ is clearly negative, so given that the Reds choose A, the Greens prefer B and complete segregation is an equilibrium. However, $\Delta\Pi^G(1,\mu_G)$ increases with α, such that for $\alpha > .5904 \equiv \hat{\alpha}$, we have that $\Delta\Pi^G(1,0) > 0$ and complete segregation is not an equilibrium anymore. In that case, there is a unique partial equilibrium.[20]

If $\alpha < .5904$ we have complete segregation as an equilibrium. In that case Proposition 2 gives us the employment rates and wages. Employment rates are given by:

$$s_A^R = s_B^G = .95 \text{ and } s_B^R = s_A^G = .9223.$$

Wages have a particular simple form in the case of complete segregation, being $w_A(1,0) = \theta\alpha$ and $w_B(1,0) = \theta(1-\alpha)$. Therefore, if we define the wage gap as $G(\mu_R,\mu_G) = 1 - w_B(\mu_R,\mu_G)/w_A(\mu_R,\mu_G)$, then the wage gap under complete segregation is $G(1,0) = 2 - 1/\alpha$. Note that at $\alpha = \hat{\alpha} = .5904$, we have

$$w_A(1,0) = 47,233 \text{ and } w_B(1,0) = 32,767$$

and the wage gap is thus $G(1,0) = .306$. Hence, a small employment gap of .9223 versus .95 is only compensated by a wage gap of 30 %!

[20] $\Delta\Pi^G(1,\mu_G)$ is not monotonically decreasing for very large α, which implies that Assumption 2 is violated. Nonetheless, there is still a unique equilibrium for all values of α.

It is worth elaborating on this potentially large wage gap. In equilibrium group R is completely specialized in education A. Therefore the wage and unemployment gap are determined by the trade off that workers from group G are making. Choosing education A gives Green workers a higher wage than education B, but in education B there would be few Green colleagues, and therefore fewer job contacts. Therefore choosing A would result in a lower employment rate for Green workers. What is surprising is that this unemployment gap may be quite small compared to the wage gap that compensates the unemployment gap. In particular, in our simulation, at $\alpha = \hat{\alpha}$, the wage gap of 30 % is compensated by an employment gap of about 3 %. The reason for this tenfold magnification is risk aversion of individuals. Individuals try to avoid the (small) risk of unemployment, in which they have a payoff equal to 0, and they are willing to accept even major losses in income in order to accomplish that.[21]

We would like to know whether an even larger wage gap can be sustained in a partial segregation equilibrium when $\alpha > \hat{\alpha} = .5904$. We therefore plot the equilibrium wages, $w_A(1, \mu^*)$ and $w_B(1, \mu^*)$, and equilibrium employments, $s_A^R(1, \mu^*)$, $s_B^R(1, \mu^*)$, $s_A^G(1, \mu^*)$ and $s_B^G(1, \mu^*)$, as function of α. Remember that the equilibrium μ^* equals zero when $\alpha \leq \hat{\alpha}$, and solves $\Delta\Pi^G(1, \mu^*) = 0$ when $\alpha > \hat{\alpha}$. These plots are shown in Figures 2 and 3.

The pictures clearly confirm Propositions 2 and 3. Moreover, for the chosen parameters we also observe that the wage gap $G(1, \mu^*)$ is maximized at $\alpha = \hat{\alpha}$. When α becomes larger than $\hat{\alpha}$, the wage of A declines and the wage of B increases until the wage gap is reversed.

We next look at the sensitivity of $\hat{\alpha}$ with respect to the parameter choices, as we saw that at $\alpha = \hat{\alpha}$ the wage gap is maximized. We do this by computing the elasticities of $\hat{\alpha}$ and of the implied wage gap $G(1, 0)$ at the chosen parameters. That is, we look at the percentage increase of $\hat{\alpha}$ and the maximum wage gap change when a parameter increases by 1% . The elasticities are shown in columns 2 and 3 of Table 2. We note that $\hat{\alpha}$ and the implied maximum wage gap are most sensitive to $\rho\theta$, the coefficient of relative risk aversion. A 1% increase in this coefficient leads to a 2% increase in the maximum wage gap. On the other hand, our calibration seems least sensitive to the network parameters $c_1(p + \kappa)$ and $c_1\lambda$. The maximum wage gap seems to be close to linear with respect to $1 - s_0$, the unemployment rate if a worker only consider direct search techniques. That is, if we chose $s_0 = .95$ instead of $s_0 = .90$, it would roughly halve the maximum wage gap.

[21] The risk aversion effect, and thus the wage gap, may be smaller if unemployment is only temporary, and individuals only care about permanent income. On the other hand, from prospect theory it is known that individual agents tend to emphasize small probabilities (Kahneman and Tversky 1979), and thus the small probability of becoming unemployed may get excessive weight in the education decision.

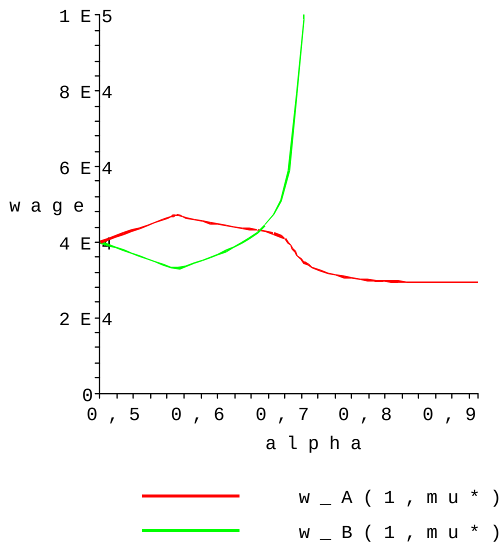

Figure 2: Equilibrium wages as function of α.

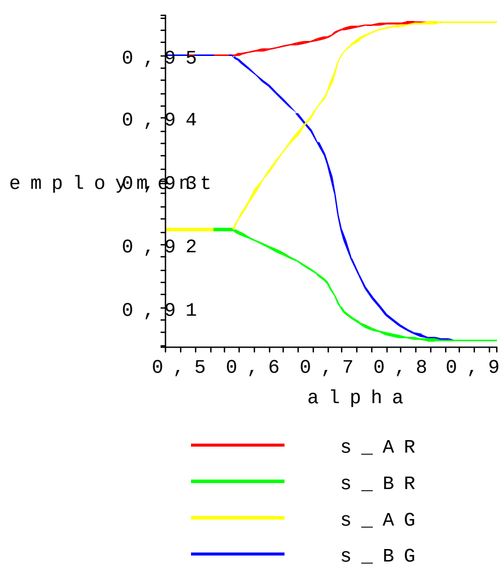

Figure 3: Equilibrium employment rates as function of α.

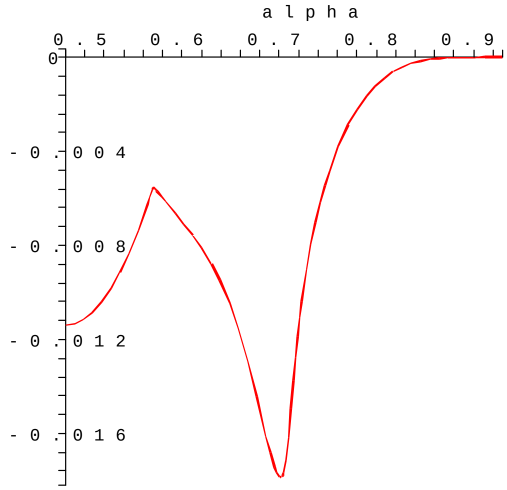

Figure 4: The percentage increase in welfare of a policy that enforces perfect integration.

5.2.2 Implications for the second-best welfare outcome

We now consider the analysis of a second-best optimum. Namely, we suppose that the government (social planner) does not have the institutions to completely control μ_R and μ_G, but that it is able to stabilize a symmetric equilibrium, such that $\mu_R = \mu_G = \mu^S$.[22] Should the government do this? In case the government stabilizes integration, we still impose the equilibrium condition, which is in this case symmetric. Therefore

$$\Pi_A^R(\mu^S, \mu^S) = \Pi_B^R(\mu^S, \mu^S) = \Pi_A^G(\mu^S, \mu^S) = \Pi_B^G(\mu^S, \mu^S).$$

Hence, in the symmetric case there is complete equality. On the other hand, in the case of segregation, we consider the equilibrium allocation $(\mu_R, \mu_G) = (1, \mu^*)$, such that Reds obtain a higher payoff than Greens. Therefore, we might face a tradeoff when assessing an integration policy. It enforces equality, but it might decrease employment.

To this purpose we plot the increase in social welfare from such an integration policy, $I = W(\mu^S, \mu^S)/W(1, \mu^*) - 1$, as function of α. Figure 4 shows this plot.

We observe that I is negative for all values of α. So for the chosen parameters the integration policy is never preferred. People are better off segregated.

[22] In the proof of Lemma 1 we show that there exists a symmetric equilibrium, but that it is unstable; that is, after a small deviation from the equilibrium individual incentives drive education choices to segregation.

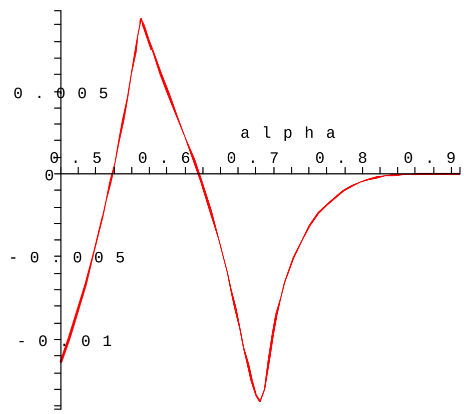

Figure 5: The percentage increase in payoffs for Green workers of a policy that enforces perfect integration.

Our results are very clear; a second best policy involves a "laissez-faire" policy, such that society becomes segregated. The intuition behind this result is twofold. First, in the case of partial segregation the equilibrium is determined by the Green workers. They trade off a benefit in wage against a loss in employment. Their individual incentives therefore already put a limit on the amount of wage inequality that can be sustained in equilibrium. Second, an integration policy would lead to lower employment rates. In a society with risk-averse individuals, society puts large emphasis on unemployment, and therefore prefers to allow for some inequality in order to obtain these higher employment rates.

We finally remark that an integration policy is only beneficial when society has *additional* distributional concerns that are not captured by the concavity of the individual utility function. For example, consider the case of a maximin social welfare function: $W_{\min} = \min_i \Pi_i$. In the integrated case, $\mu_R = \mu_G = \mu^S$, everyone obtains the same payoff, whereas in the segregated case workers from group G are worse off. Therefore, $W_{\min}(1, \mu^*) = \Pi_B^G(1, \mu^*)$ and $W_{\min}(\mu^S, \mu^S) = \Pi_B^G(\mu^S, \mu^S)$. We show a comparison of these two payoffs, $\Pi_B^G(\mu^S, \mu^S)/\Pi_B^G(1, \mu^*) - 1$, in Figure 5.

We observe that the Green workers would benefit from an integration policy for values of α around $\hat{\alpha}$, where the wage gap is particularly large. In such a case, strong distributional concerns would justify integration.

6 Summary and conclusions

We have investigated a simple social network framework where jobs are obtained through a network of contacts formed stochastically, after career decisions had been made. We have established that even with a very small amount of homophily within each social group, stable occupational segregation equilibria will arise. If the wage differential across the occupations is not too large, complete segregation will always be sustainable. If the wage differential is large, complete segregation cannot be sustained, but a partial segregation equilibrium in which one of the group fully specializes in one education while the other group mixes over the career tracks, is sustainable. Furthermore, our model is able to explain sustained unemployment and wage differences between the social groups.

We also analyze the implications of our model from a social planner's point of view. In the first best social welfare optimum, we find that segregation is the socially preferred outcome. Subject to proper calibration of our model parameters, a second best social welfare analysis supports a laissez-faire policy, where society also becomes segregated, shaped by individual incentives. Both these conclusions are valid in light of 'reasonable' concavity features of the individual utility function. Our social welfare conclusions cast some doubts on an "always integration" policy choice; if job referrals through contact networks are relevant in matching workers to vacancies, and if the mechanisms of our model are the correct ones, an integration approach would only be justified under strong additional distributional concerns, not reflected in the individual utility functions.

We assume the probability that an individual i forms a tie with individual j to be exogenously given and constant. In practice, establishing a friendship between two individuals typically involves rational decision making.

We assumed that individuals first choose an education, and then form a network of job contacts. As a consequence, individuals have to make *expectations* about the network they could form, and base their education decisions on these expectations. This is in contrast to earlier work on the role of networks in the labor market. In former research, the network was supposed to be already in place, or the network was formed in the first stage (Montgomery 1991, Calvó-Armengol 2004, Calvó-Armengol and Jackson 2004).

Our departure from the earlier frameworks raises questions about the assumed timing of the education choice. Are crucial career decisions made before or after job contacts are formed? One might be tempted to answer: both. Of course everyone is born with family ties, and in early school and in the neighborhood children form more ties. It is also known that peer-group pressure among children has a strong effect on decisions to, for instance, smoke or engage in criminal activities and, no doubt, family and early friends do form a non-negligible source of influence when making crucial career decisions. However, we argue that most *job-relevant contacts* (the so called 'instrumental ties') are made later, for instance at the university, or early at the workplace, hence after a specialized career track had been chosen. In spite of the fact that those ties are typically not as strong as family ties, they are more likely to provide relevant information on vacancies to job seekers; Granovetter (1973, 1985) provides convincing evidence that job seekers more often receive crucial job information from acquaintances ("weak ties"), rather than from family or very close friends ("strong ties"). If the vast majority of such instrumental ties are formed after the individual embarked on a (irreversible) career, then it is justified to consider a model in which the job contact network is formed after making a career choice.

While our social interaction model can describe empirical patterns of occupational segregation and wage inequality between gender, racial or ethnical groups, other factors are also documented to play a significant role in this context. This model should thus be seen as complement to alternatives, such as taste discrimination or rational bias by employers, which are still present in the market despite their (predicted) erosion over time, due to both competitive pressure and institutional instruments. It is therefore pertinent to directly investigate in future research how relevant are the mechanisms described in this paper and to assess their relative strength in explaining observed occupational segregation, vis-à-vis other proposed theories.

Our model easily allows for interesting extensions. One avenue for future research is to extend our framework to issues such as the position of minority versus majority groups, by modeling the interaction between social groups of unequal sizes. Another avenue is to consider heterogeneity in productivity. This would allow us to analyze the mismatch of workers to firms due to network effects. We intend to pursue these lines of research in the future.

A Proofs

The proof of Proposition 1 uses the following lemma:

Lemma 1 *Suppose Assumptions 1 and 2 hold. A weakly stable equilibrium* (μ_R^*, μ_G^*)*, in which* $0 < \mu_R^* < 1$ *and* $0 < \mu_G^* < 1$*, does not exist.*

Proof. Suppose (μ_R^*, μ_G^*) is a stable equilibrium, and $\mu_R^* \in (0,1)$ and $\mu_G^* \in (0,1)$. By condition (2)

$$\Pi_A^R(\mu_R^*, \mu_G^*) = \Pi_B^R(\mu_R^*, \mu_G^*) \text{ and } \Pi_A^G(\mu_R^*, \mu_G^*) = \Pi_B^G(\mu_R^*, \mu_G^*) \tag{18}$$

Substituting (8)-(9) into (18) and rewriting, these equations become

$$\frac{U(w_A(\mu_R^*, \mu_G^*))}{U(w_B((\mu_R^*, \mu_G^*))} = \frac{s_B^R(\mu_R^*, \mu_G^*)}{s_A^R(\mu_R^*, \mu_G^*)} = \frac{s_B^G(\mu_R^*, \mu_G^*)}{s_A^G(\mu_R^*, \mu_G^*)}. \tag{19}$$

Since $\lambda > 0$, $\mu_R^* > \mu_G^*$ implies $s_A^R > s_A^G$ and $s_B^R < s_B^G$. But this means that if $\mu_R^* > \mu_G^*$, then

$$\frac{s_B^R(\mu_R^*, \mu_G^*)}{s_A^R(\mu_R^*, \mu_G^*)} < \frac{s_B^G(\mu_R^*, \mu_G^*)}{s_A^G(\mu_R^*, \mu_G^*)}.$$

which contradicts (19). The same reasoning holds for $\mu_R^* < \mu_G^*$. Hence, it must be that $\mu_R^* = \mu_G^*$.

However (μ_R^*, μ_G^*) with $\mu_R^* = \mu_G^*$ cannot be a stable equilibrium. To see this, suppose that (μ^*, μ^*) with $\mu^* \in (0,1)$ is a stable equilibrium. Hence $\Pi_A^X(\mu^*, \mu^*) = \Pi_B^X(\mu^*, \mu^*)$ for $X \in \{R, G\}$ and $\frac{\partial \Delta\Pi^X}{\partial \mu_X} < 0$ at $\mu_R = \mu_G = \mu^*$, and $\det(G(\mu^*, \mu^*) > 0$, where $G(\mu) = D\Delta\Pi(\mu)$ is the Jacobian of $\Delta\Pi \equiv (\Delta\Pi^R, \Delta\Pi^G)$ with respect to $\mu \equiv (\mu_R, \mu_G)$.

Since $\lambda > 0$, it must be that

$$\frac{\partial s_A^X}{\partial \mu_X} > \frac{\partial s_A^X}{\partial \mu_Y} > 0 \tag{20}$$

and

$$\frac{\partial s_B^X}{\partial \mu_X} < \frac{\partial s_B^X}{\partial \mu_Y} < 0 \tag{21}$$

for $X, Y \in \{R, G\}$ and $Y \neq X$. Furthermore, if $\mu_R = \mu_G = \mu^*$, then $s_A^X = s_A^Y$, $\frac{\partial L_A}{\partial \mu_X} = \frac{\partial L_A}{\partial \mu_Y}$, $\frac{\partial L_B}{\partial \mu_X} = \frac{\partial L_B}{\partial \mu_Y}$, and therefore,

$$\frac{\partial w_A}{\partial \mu_X} = \frac{\partial w_A}{\partial \mu_Y} \tag{22}$$

and

$$\frac{\partial w_B}{\partial \mu_X} = \frac{\partial w_B}{\partial \mu_Y}. \tag{23}$$

From (20)-(23) and Assumption 2, it follows that, at $\mu_R = \mu_G = \mu^*$,

$$\frac{\partial \Delta \Pi^X}{\partial \mu_Y} < \frac{\partial \Delta \Pi^X}{\partial \mu_X} < 0.$$

for $X, Y \in \{R, G\}$, $X \neq Y$. But then it is straightforward to see that $\det(G(\mu^*, \mu^*)) < 0$. This contradicts stability. ∎

Proof of Proposition 1. (i) If (10) holds, then

$$\Pi_A^R(1,0) > \Pi_B^R(1,0) \text{ and } \Pi_A^G(1,0) < \Pi_B^G(1,0).$$

Hence, $(\mu_R, \mu_G) = (1,0)$ is clearly a stable equilibrium. The same is true for $(\mu_R, \mu_G) = (0,1)$. Lemma 1 and Assumption 2 ensure that these are the only two equilibria.

(ii) If (11) is true, then

$$\Pi_A^G(1,0) > \Pi_B^G(1,0). \tag{24}$$

Furthermore, from Assumption 1 we know that $\frac{\partial \Delta \Pi^G(1,\mu_G)}{\partial \mu_G} < 0$ for all $\mu_G \in [0,1]$. It follows from Assumption 1, equation (24) and continuity of F, U and s, that there must be a unique μ^*, such that

$$\Pi_A^G(1,\mu^*) = \Pi_B^G(1,\mu^*).$$

Moreover, $s_A^R(1,\mu^*) > s_A^G(1,\mu^*)$ and $s_B^G(1,\mu^*) > s_B^R(1,\mu^*)$, so we have at $(\mu_R, \mu_G) = (1, \mu^*)$

$$\Pi_A^X > \Pi_B^Y = \Pi_A^Y > \Pi_B^X. \tag{25}$$

It is therefore clear that $(\mu_R, \mu_G) = (1, \mu^*)$ is a stable equilibrium. The same is true for $(\mu_R, \mu_G) = (\mu^*, 1)$.

To show that there is no other equilibrium, note that by (11) $\Pi_A^R(1,0) > \Pi_B^R(1,0)$. Assumption 2 then implies that $\Pi_A^R(\mu,0) > \Pi_B^R(\mu,0)$ for all $\mu \in [0,1]$. Hence, $(\mu,0)$ and, similarly, $(0,\mu)$ cannot be an equilibrium. By Lemma 1 we also know that there is no mixed equilibrium. ∎

Proof of Proposition 2. The equations follow almost directly. We have

$$s_A^R(1,0) = s_B^G(1,0) = s_H > s_L = s_B^R(1,0) = s_A^G(1,0).$$

Further, by assumption $w_A \geq w_B$ at $(\mu_R, \mu_G) = (1,0)$. Finally, at $(\mu_R, \mu_G) = (1,0)$

$$U(w_A)s_A^R \geq U(w_B)s_B^G \geq U(w_A)s_A^G \geq U(w_B)s_B^R,$$

and this is equivalent to (12). ∎

Proof of Proposition 3. Consider the stable equilibrium at $(1, \mu^*)$. Since it is an equilibrium we know that

$$\Pi_A^G(1, \mu^*) = \Pi_B^G(1, \mu^*).$$

In the proof of Proposition 1, equation (25), we already demonstrated the inequality (14) Further, by Assumption 2 we know that $\Delta\Pi^G(1, \mu_G)$ is strictly monotonically decreasing in μ_G.

(i) If $\hat{\mu} < \frac{\lambda}{2(p+\kappa+\lambda)}$, then $s_A^G(1, \hat{\mu}) < s_B^G(1, \hat{\mu})$. As $w_A(1, \hat{\mu}) = w_B(1, \hat{\mu})$ it must be that

$$\Pi_A^G(1, \hat{\mu}) < \Pi_B^G(1, \hat{\mu}).$$

But then it also must be that $\mu^* < \hat{\mu}$. As we consider a partial equilibrium, we know that $\mu^* > 0$. Hence, $0 < \mu^* < \hat{\mu}$ and $w_A(1, \hat{\mu^*}) > w_B(1, \hat{\mu^*})$, as $w_A(\mu_R, \mu_G)$ is a decreasing function, whereas $w_B(\mu_R, \mu_G)$ is increasing.

(ii) If $\hat{\mu} > \frac{\lambda}{2(p+\kappa+\lambda)}$, then $s_A^G(1, \hat{\mu}) > s_B^G(1, \hat{\mu})$ and $\Pi_A^G(1, \hat{\mu}) < \Pi_B^G(1, \hat{\mu})$. But then $\mu^* > \hat{\mu}$. By Assumption 1 we know that $\mu^* < 1$. Hence, $\hat{\mu} < \mu^* < 1$, and therefore $w_A(1, \hat{\mu^*}) < w_B(1, \hat{\mu^*})$ ■

We next continue with the proof of Proposition 4. This proof uses the following lemma:

Lemma 2 *Suppose that for all $x \in [0, (p + \kappa + \lambda)/2]$*

$$s''(x) > -\frac{4}{\lambda}\, s'(x). \tag{26}$$

(i) If $\mu_X > \mu_Y$ for $X, Y \in \{R, G\}$, then

$$\frac{\partial L_A}{\partial \mu_X}(\mu_R, \mu_G) > \frac{\partial L_A}{\partial \mu_Y}(\mu_R, \mu_G) > 0, \tag{27}$$

and

$$\frac{\partial L_B}{\partial \mu_Y}(\mu_R, \mu_G) < \frac{\partial L_B}{\partial \mu_X}(\mu_R, \mu_G) < 0. \tag{28}$$

(ii) If $\mu_R = \mu_G = \mu$, then

$$\frac{\partial^2 L_A}{(\partial \mu_X)^2}(\mu, \mu) > \frac{\partial^2 L_A}{\partial \mu_X \partial \mu_Y}(\mu, \mu), \tag{29}$$

and

$$\frac{\partial^2 L_B}{(\partial \mu_X)^2}(\mu, \mu) > \frac{\partial^2 L_B}{\partial \mu_X \partial \mu_Y}(\mu, \mu). \tag{30}$$

Proof. (i) It is easy to derive that for $X \in \{R, G\}$:

$$\frac{\partial L_A}{\partial \mu_X} = \frac{1}{2}\left(s_A^X + \mu_R \frac{\partial s_A^R}{\partial \mu_X} + \mu_G \frac{\partial s_A^G}{\partial \mu_X}\right) > 0 \tag{31}$$

$$\frac{\partial L_B}{\partial \mu_X} = \frac{1}{2}\left(-s_B^X + (1-\mu_R)\frac{\partial s_B^R}{\partial \mu_X} + (1-\mu_G)\frac{\partial s_B^G}{\partial \mu_X}\right) < 0 \tag{32}$$

at (μ_R, μ_G). From (31) and (32), we find that for all $X, Y \in \{R, G\} : \partial L_A/\partial \mu_X > \partial L_A/\partial \mu_Y$ is equivalent to

$$s_A^X + \mu_X\left(\frac{\partial s_A^X}{\partial \mu_X} - \frac{\partial s_A^X}{\partial \mu_Y}\right) > s_A^Y + \mu_Y\left(\frac{\partial s_A^Y}{\partial \mu_Y} - \frac{\partial s_A^Y}{\partial \mu_X}\right). \tag{33}$$

With the definition of s_A^X in (4) we can write out

$$s_A^X + \mu_X\left(\frac{\partial s_A^X}{\partial \mu_X} - \frac{\partial s_A^X}{\partial \mu_Y}\right) = s\left((p+\kappa)\bar{\mu} + \lambda\mu_X/2\right) + \frac{\mu_X\lambda}{2}s'\left((p+\kappa)\bar{\mu} + \lambda\mu_X/2\right) \tag{34}$$

when $X \neq Y$. Therefore $\mu_X > \mu_Y$ is equivalent to (33), whenever (34) is strictly monotone increasing with μ_X, where we can treat $\bar{\mu} = (\mu_X + \mu_Y)/2$ as a constant. It is easy to check that this is indeed the case under condition (26). We conclude that hypothesis (27) holds whenever $\mu_X > \mu_Y$. With a similar derivation one can show that condition (26) implies (28) as well.

(ii) The second derivatives of L_A and L_B with respect to μ_X and μ_Y are

$$\frac{\partial^2 L_A}{\partial \mu_X \partial \mu_Y} = \frac{1}{2}\left(\frac{\partial s_A^X}{\partial \mu_Y} + \frac{\partial s_A^Y}{\partial \mu_X} + \mu_R \frac{\partial^2 s_A^R}{\partial \mu_X \partial \mu_Y} + \mu_G \frac{\partial^2 s_A^G}{\partial \mu_X \partial \mu_Y}\right) \tag{35}$$

$$\frac{\partial^2 L_B}{\partial \mu_X \partial \mu_Y} = \frac{1}{2}\left(-\frac{\partial s_B^X}{\partial \mu_Y} - \frac{\partial s_B^Y}{\partial \mu_X} + (1-\mu_R)\frac{\partial^2 s_B^R}{\partial \mu_X \partial \mu_Y} + (1-\mu_G)\frac{\partial^2 s_B^G}{\partial \mu_X \partial \mu_Y}\right). \tag{36}$$

Taking the second derivatives of s_A^X, evaluating at $\mu_R = \mu_G = \mu$ and reordering, we get that (29) is equivalent to

$$s''((p+\kappa+\lambda)\mu/2) < -\frac{4}{\lambda\mu}s'((p+\kappa+\lambda)\mu/2). \tag{37}$$

Inequality (37) clearly holds if condition (26) holds, which proves (29). In a similar fashion, (26) implies (30) ■

Proof of Proposition 4. Suppose that $W(\mu_R, \mu_G)$ is maximized at $(\mu_R, \mu_G) = (\tilde{\mu}_R, \tilde{\mu}_G)$, where $\tilde{\mu}_R \in (0,1)$ and $\tilde{\mu}_G \in (0,1)$. Define $c \equiv L_A(\tilde{\mu}_R, \tilde{\mu}_G)/L_B(\tilde{\mu}_R, \tilde{\mu}_G)$, and consider the constrained maximization problem:

$$\max_{\mu_R \in [0,1], \mu_G \in [0,1]} W(\mu_R, \mu_G) \text{ s.t. } L_A(\mu_R, \mu_G) = cL_B(\mu_R, \mu_G). \tag{38}$$

Because by definition of c, the solution $(\tilde{\mu}_R, \tilde{\mu}_G)$ satisfies the restriction

$$g(\mu_R, \mu_G) = cL_B(\mu_R, \mu_G) - L_A(\mu_R, \mu_G) = 0, \tag{39}$$

it actually solves the maximization problem (38).

Define the feasible set $C = \{\mu_R \in [0,1], \mu_G \in [0,1] | g(\mu_R, \mu_G) = 0\}$. By the assumption of constant returns to scale, we have that for all $(\mu_R, \mu_G) \in C$: $w_A(\mu_R, \mu_G)$ and $w_B(\mu_R, \mu_G)$ are constant, and therefore, at all $(\mu_R, \mu_G) \in C$, the welfare function (16) can be written as

$$W(\mu_R, \mu_G) = L_A(\mu_R, \mu_G)(U(\overline{w_A}) + U(\overline{w_B})/c),$$

which is monotone increasing with $L_A(\mu_R, \mu_G)$. Therefore, the solution $(\tilde{\mu}_R, \tilde{\mu}_G)$ also solves the following maximization problem:

$$\max_{\mu_R \in [0,1], \mu_G \in [0,1]} L_A(\mu_R, \mu_G) \text{ s.t. } L_A(\mu_R, \mu_G) = cL_B(\mu_R, \mu_G). \tag{40}$$

We verify that $(\tilde{\mu}_R, \tilde{\mu}_G)$ indeed satisfy the first- and second-order conditions of problem (40). The Lagrangian is given by

$$\mathcal{L}(\mu_R, \mu_G, \psi) = (1-\psi)L_A(\mu_R, \mu_G) + \psi c L_B(\mu_R, \mu_G).$$

Since $(\tilde{\mu}_R, \tilde{\mu}_G)$ is supposed to be interior, the following first order constraints should hold:

$$\frac{\partial \mathcal{L}}{\partial \mu_R}(\tilde{\mu}_R, \tilde{\mu}_G, \psi) = (1-\psi)\frac{\partial L_A}{\partial \mu_R}(\tilde{\mu}_R, \tilde{\mu}_G) + \psi \frac{\partial L_B}{\partial \mu_R}(\tilde{\mu}_R, \tilde{\mu}_G) = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_G}(\tilde{\mu}_R, \tilde{\mu}_G, \psi) = (1-\psi)\frac{\partial L_A}{\partial \mu_G}(\tilde{\mu}_R, \tilde{\mu}_G) + \psi \frac{\partial L_B}{\partial \mu_G}(\tilde{\mu}_R, \tilde{\mu}_G) = 0. \tag{42}$$

The first part of Lemma 2 implies that $\psi \in (0,1)$ and that under condition (26): $\mu_R > \mu_G$ if and only if $\partial \mathcal{L}/\partial \mu_R > \partial \mathcal{L}/\partial \mu_G$. Therefore, condition (26) and the first-order conditions imply that $\tilde{\mu}_R = \tilde{\mu}_G \equiv \tilde{\mu}$.

Since $\tilde{\mu}_R = \tilde{\mu}_G$ defines a unique point in C, the second-order condition should hold at $\tilde{\mu}_R = \tilde{\mu}_G$, which says that the Hessian of the Lagrangian with respect to (μ_R, μ_G) evaluated at the social optimum, $D^2_{\mu_R,\mu_G}\mathcal{L}(\tilde{\mu}, \tilde{\mu}, \psi)$, is negative definite on the subspace $\{z_R, z_G | z_R(\partial g/\partial \mu_R) + z_G(\partial g/\partial \mu_G) = 0\}$. The second order condition is thus that at $(\mu_R, \mu_G) = (\tilde{\mu}, \tilde{\mu})$:

$$2\frac{\partial g}{\partial \mu_R}\frac{\partial g}{\partial \mu_G}\frac{\partial^2 \mathcal{L}}{\partial \mu_R \partial \mu_G} - \left(\frac{\partial g}{\partial \mu_R}\right)^2 \frac{\partial^2 \mathcal{L}}{(\partial \mu_G)^2} - \left(\frac{\partial g}{\partial \mu_G}\right)^2 \frac{\partial^2 \mathcal{L}}{(\partial \mu_R)^2} > 0. \tag{43}$$

Because $\frac{\partial g}{\partial \mu_R}(\tilde{\mu}, \tilde{\mu}) = \frac{\partial g}{\partial \mu_G}(\tilde{\mu}, \tilde{\mu})$, and $\frac{\partial^2 \mathcal{L}}{(\partial \mu_G)^2}(\tilde{\mu}, \tilde{\mu}) = \frac{\partial^2 \mathcal{L}}{(\partial \mu_R)^2}(\tilde{\mu}, \tilde{\mu})$, the second order condition (43) simplifies to $\frac{\partial^2 \mathcal{L}}{\partial \mu_R \partial \mu_G}(\tilde{\mu}, \tilde{\mu}) > \frac{\partial^2 \mathcal{L}}{(\partial \mu_R)^2}(\tilde{\mu}, \tilde{\mu})$, or equivalently

$$(1-\psi)\frac{\partial^2 L_A}{\partial \mu_R \partial \mu_G}(\tilde{\mu}, \tilde{\mu}) + \psi \frac{\partial^2 L_B}{\partial \mu_R \partial \mu_G}(\tilde{\mu}, \tilde{\mu}) > (1-\psi)\frac{\partial^2 L_A}{(\partial \mu_R)^2}(\tilde{\mu}, \tilde{\mu}) + \psi \frac{\partial^2 L_B}{(\partial \mu_R)^2}(\tilde{\mu}, \tilde{\mu}). \tag{44}$$

By the second part of Lemma 2, inequality (44) cannot hold under condition (26). Therefore we have a contradiction and the non-segregation allocation $(\tilde{\mu}_R, \tilde{\mu}_G)$ cannot be a social optimum. Since a social optimum exists by continuity of W and compactness of $[0, 1]^2$, the social optimum necessarily has to involve complete or partial segregation. ■

References

[1] Albelda, R.P., "Occupational Segregation by Race and Gender: 1958–1981," *Industrial and Labour Relations Review* 39 (April 1986), 404-411.

[2] Alesina, A. and E. La Ferrara, "Participation in Heterogeneous Communities," *Quarterly Journal of Economics* 115 (August 2000), 847-904.

[3] Alesina, A. and E. La Ferrara, "Who Trusts Others?," *Journal of Public Economics* 85 (2002), 207-234.

[4] Altonji, J.G. and R.M. Blank, "Race and Gender in the Labor Market," in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, Volume 3 (Amsterdam: Elsevier, 1999), 3143-3259.

[5] Arrow, K.J., "What Has Economics to Say about Racial Discrimination?," *Journal of Economic Perspectives* 12 (Spring 1998), 91-100.

[6] Beller, A.H., "Occupational Segregation by Sex: Determinants and Changes," *Journal of Human Resources* 17 (Summer 1982), 371-392.

[7] Benabou, R., "Workings of a City: Location, Education, and Production," *Quarterly Journal of Economics*, 108 (August 1993), 619-652.

[8] Bewley, T.F., *Why Wages Don't Fall During a Recession*, (Cambridge MA: Harvard University Press, 1999).

[9] Bramoullé, Y. and G. Saint-Paul, "Social Networks and Labour Market Transitions", *Labour Economics*,17 (January 2010), 188-195.

[10] Brass, D.J., "Men's and Women's Networks: A Study of Interaction Patterns and Influence in an Organization," *Academy of Management Journal* 28 (June 1985), 327-343.

[11] Calvó-Armengol, A., "Job Contact Networks," *Journal of Economic Theory* 115 (2004), 191-206.

[12] Calvó-Armengol, A. and M.O. Jackson, "The Effects of Social Networks on Employment and Inequality," *American Economic Review*, 94 (June 2004), 426-454.

[13] Calvó-Armengol, A. and M.O. Jackson, "Networks in Labor Markets: Wage and Employment Dynamics and Inequality," *Journal of Economic Theory*, 132 (January 2007), 27-46.

[14] Calvó-Armengol, A. and Y. Zenou, "Job Matching, Social Network and Word-of-Mouth Communication," *Journal of Urban Economics*, 57 (2005), 500-522.

[15] Charles, M. and D.B. Grusky, *Occupational Ghettos: The Worldwide Segregation of Women and Men* (Stanford CA: Stanford University Press, 2004).

[16] Cohen, A. and L. Einav, "Estimating Risk Preferences from Deductible Choices," *American Economic Review*, 97 (2007), 745-788.

[17] Currarini, S., M.O. Jackson and P. Pin, "An Economic Model of Friendship: Homophily, Minorities and Segregation," *Econometrica*, 77 (2009), 1003-1045.

[18] Datcher, L., "The Impact of Informal Networks on Quit Behavior," *Review of Economics and Statistics*, 65 (August 1983), 491-495.

[19] Datcher Loury, L., "Some Contacts Are More Equal than Others: Informal Networks, Job Tenure, and Wages," *Journal of Labour Economics*, 24 (April 2006), 299-318.

[20] Dates, J., "Race, Racial Attitudes and Adolescent Perceptions of Black Television Characters," *Journal of Broadcasting*, 24 (Fall 1980), 549-560.

[21] Devine, T.J. and N.M. Kiefer, *Empirical Labor Economics: The Search Approach* (New York: Oxford University Press, 1991).

[22] Doeringer P.B. and M.J. Piore, *Internal Labor Markets and Manpower Analysis* (Lexington: Heath Lexington Books, 1971).

[23] Fernandez, R.M. and M.L. Sosa, "Gendering the job: Networks and Recruitment at a Call Centre," *American Journal of Sociology*, 111 (November 2005), 859-904.

[24] Fisman, R., S. Iyengar, E. Kamenica, I. Simonson, "Gender Differences in Mate Selection: Evidence from a Speed Dating Experiment," *Quarterly Journal of Economics*, 121 (May 2006), 673-697.

[25] Gertner, R., "Game Shows and Economic Behavior: Risk Taking on “Card Sharks”," *Quarterly Journal of Economics*, 108 (May 1993), 507-521.

[26] Granovetter, M.S., "The Strength of Weak Ties," *American Journal of Sociology*, 78 (May 1973), 1360–1380.

[27] Granovetter, M.S., *Getting a Job: A Study of Contacts and Careers*, 2nd edition (Chicago: University of Chicago Press, 1995).

[28] Granovetter, M.S., "A Theoretical Agenda for Economic Sociology," in M.F. Guillén, R. Collins, P. England and M. Meyer, eds., *The New Economic Sociology: Developments in an Emerging Field* (New York: Russell Sage Foundation, 2002), 35-59.

[29] Holzer, H.J., "Informal Job Search and Black Youth Unemployment," *American Economic Review*, 77 (June 1987), 446-452.

[30] Hraba, J. and G. Grant, "Black is Beautiful: A Reexamination of Racial Preference and Identification," *Journal of Personality and Social Psychology*, 16 (November 1970), 398-402.

[31] Ibarra, H., "Homophily and Differential Returns: Sex Differences in Network Structure and Access in an Advertising Firm," *Administrative Science Quarterly*, 37 (September 1992), 422-447.

[32] Ioannides, Y.M. and L. Datcher Loury, "Job Information Networks, Neighborhood Effects, and Inequality," *Journal of Economic Literature*, 42 (December 2004), 1056–1093.

[33] Ioannides, Y.M. and A.R. Soetevent, "Wages and Employment in a Random Social Network with Arbitrary Degree Distribution," *American Economic Review Papers and Proceedings*, 96 (May 2006), 270-274.

[34] Kahneman, D., and A. Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47 (March 1979), 263-292.

[35] King, M.C., "The Evolution of Occupational Segregation by Race and Gender, 1940-1988," *Monthly Labor Review*, 115 (April 1992), 30-36.

[36] Lazarsfeld, P.F. and R.K. Merton, "Friendship as Social Process: A Substantive and Methodological Analysis," in M. Berger, T. Abel and C. Page, eds., *Freedom and Control in Modern Society* (Princeton NJ: Van Nostrand, 1954), 18-66.

[37] Lee, J., "American Idol: Evidence of Same-Race Preferences?," IZA Discussion paper No. 1974, IZA, Germany, February 2006.

[38] Marsden, P.V., "Core Discussion Networks of Americans," *American Sociological Review*, 52 (February 1987), 122-131.

[39] Marsden, P.V., "Homogeneity in Confiding Relations," *Social Networks*, 10 (March 1988), 57-76.

[40] Mayer, A. and S.L. Puller, "The Old Boy (and Girl) Network: Social Network Formation on University Campuses," *Journal of Public Economics*, 92 (February 2008), 329-347.

[41] McPherson, M., L. Smith-Lovin and J.M. Cook, "Birds of a Feather: Homophily in Social Networks," *Annual Review of Sociology*, 27 (2001), 415-444.

[42] Metrick, A., "A Natural Experiment in “Jeopardy!”," *American Economic Review*, 85 (March 1995), 240-253.

[43] Montgomery, J.D., "Social Networks and Labour-Market Outcomes: Toward an Economic Analysis," *American Economic Review*, 81 (December 1991), 1408-1418.

[44] Montgomery, J.D., "Social Networks and Persistent Inequality in the Labour Market," mimeo, Northwestern University, 1992.

[45] Padavic, B.F. and I. Reskin, *Women and Men at Work*, 2nd edition (Thousand Oaks: Pine Forge Press, 2002).

[46] Pettit, B. and J. Hook, "The Structure of Women's Employment in Comparative Perspective", *Social Forces*, 84 (December 2005), 779-801.

[47] Rees, A., "Information Networks in Labor Markets," *American Economic Review*, 56(March-May 1966), 559-566.

[48] Schelling, T.C., "Dynamic Models of Segregation," *Journal of Mathematical Sociology*, 1 (1971), 143-186.

[49] Shrum, W., N.H. Cheek and S.M. Hunter, "Friendship in School: Gender and Racial Homophily," *Sociology of Education*, 61 (October 1988), 227-239.

[50] Simon, C.J. and J.T. Warner, "Matchmaker, Matchmaker: The Effect of Old Boy Networks on Job Match Quality, Earnings, and Tenure," *Journal of Labor Economics*, 10 (July 1992), 306-330.

[51] Sørensen, J.B., "The Organizational Demography of Racial Employment Segregation," *American Journal of Sociology*, 110 (November 2004), 626-671.

[52] Tampubolon, G., "Fluxes and Constants in the Dynamics of Friendships," ESRC Research Methods Programme Working Paper No. 25, University of Manchester, 2005.

[53] Topa, G., "Social Interactions, Local Spillovers and Unemployment," *Review of Economic Studies*, 68 (2001), 261-295.

[54] Weeden, K.A. and J.B. Sørensen, "Two-Dimensional Models of Sex Segregation: Industries and Occupations," in M. Charles and D.B. Grusky, eds., *Occupational Ghettos: The Worldwide Segregation of Women and Men* (Stanford: Stanford University Press, 2004), 245-294.

[55] Wong, L.Y., "Why Do Only 5.5% of Black Men Marry White Women?," *International Economic Review*, 44 (August 2003), 803-826.