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# SUPPLEMENT TO "RETURNS TO TENURE OR SENIORITY?": ADDITIONAL TABLES AND ESTIMATIONS

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We provide this supplement to the paper "Returns to Tenure or Seniority?". It includes information on the residual autocovariances for within-job log wage innovations, in Appendix A, which we use to compute the standard deviation of the permanent shocks. In Appendix B, we show how one can estimate $\beta_1$ and $\beta_2$ for both the standard Topel and the Topel variant with spell fixed effects specifications, when the model includes time dummy variables.

## APPENDIX A: ADDITIONAL TABLES

AN MA(1) PROCESS MADE UP of a mixture of permanent and transitory shocks well describes the autocovariance patterns in Table VII. We perform a back-of-the-envelope computation of the standard deviation of the permanent shocks. Let $q_{ijt}$ and $u_{ijt}$ be the transitory and permanent shock, respectively. Then $\Delta v_{ijt} = u_{ijt} + q_{ijt} - q_{ij,t-1}$. Hence, $\text{Var}(\Delta v_{ijt}) = \text{Var}(u_{ijt}) + 2\,\text{Var}(q_{ijt})$ and $\text{Cov}(\Delta v_{ijt}, \Delta v_{ij,t-1}) = -\text{Var}(q_{ijt})$, so that

$$\text{Var}(u_{ijt}) = \text{Var}(\Delta v_{ijt}) + 2\,\text{Cov}(\Delta v_{ijt}, \Delta v_{ij,t-1}).$$

We obtain 0.10 for Denmark and 0.12 for Portugal as standard deviation of the permanent shocks, which is in line with earlier estimates obtained for the United States. The following Table VIII replicates Topel's (1991) Table 4, Panel B, using Topel wage growth regressions with seniority index included—corresponding to Table III, column II in our main text—on various remaining job durations. Our conclusion is the same as Topel's for this type of exercise.

## APPENDIX B: ESTIMATION OF $\beta_1$ AND $\beta_2$ FOR STANDARD AND FE TOPEL, WHEN INCLUDING TIME DUMMY VARIABLES

### B.1. *Topel's Model*

Consider our empirical model as discussed in the main text of the paper:

$$\log w_{ijt} = \beta_0 + \beta_{11} X_{ijt} + \beta_{12} X_{ijt}^2 + \beta_{21} T_{ijt} + \beta_{22} T_{ijt}^2 + \varepsilon_{ijt},$$

where we omit higher-order terms in $X_{ijt}$ and $T_{ijt}$, as well as $r_{ijt}$ and $n_{jt}$, for convenience. Taking within-spell first-differences, this results in

$$\begin{aligned} (11)\qquad \Delta \log w_{ijt} &= \beta_{11} + \beta_{21} + \beta_{12}\Delta X_{ijt}^2 + \beta_{22}\Delta T_{ijt}^2 + \Delta\tau_t + \Delta\nu_{ijt} \\ &= \beta_{11} + \beta_{21} + \tau_2 + \beta_{12}\Delta X_{ijt}^2 + \beta_{22}\Delta T_{ijt}^2 + \delta_t + \Delta\nu_{ijt}, \end{aligned}$$

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TABLE VII

RESIDUAL AUTOCOVARIANCES FOR WITHIN-JOB LOG WAGE INNOVATIONS[a]

| Lag | Denmark 1980–2001 | Portugal 1986–2009 |
|---|---|---|
| 0 | 0.0195 | 0.0355 |
| | (0.00002) | (0.00007) |
| 1 | −0.0043 | −0.0108 |
| | (0.00001) | (0.00005) |
| 2 | −0.0004 | −0.0009 |
| | (0.00001) | (0.00002) |
| 3 | −0.0002 | −0.0005 |
| | (0.00001 | (0.00003) |
| 4 | −0.0003 | 0.00001 |
| | (0.00001) | (0.00003) |
| 5 | −0.00002 | −0.00002 |
| | (0.00001) | (0.00003) |
| 6 | −0.00009 | −0.0005 |
| | (0.00001) | (0.00003) |
| Number of observations | 8,902,997 | 9,884,371 |

[a]The generating regressions are the Topel regressions with seniority index included (see Table III in the paper). Only the first six lags are displayed here. Standard errors in parentheses.

where we define, for $t \geq 3$,

$$\delta_t = \Delta\tau_t - \tau_2,$$

or, by using repeated substitution, we obtain

$$\tau_t = (t-1)\tau_2 + \sum_{s=3}^{t} \delta_s. \tag{12}$$

Equation (11) shows that a regression of $\Delta \log w_{ijt}$ on $\Delta X_{ijt}^2$, $\Delta T_{ijt}^2$ and a full set of time dummies yields consistent estimates of $B = \beta_{11} + \beta_{21} + \tau_2$, $\beta_{12}$, and $\beta_{22}$. Substituting (12) into (11), using $X_{ijt} = X_{ij,0} + T_{ijt}$ as well as $t = t_{ij,0} + T$, where $X_{0,ij}$ and $t_{ij,0}$ are respectively experience and time at the start of the spell, and replacing the coefficients by their estimates, we obtain

$$\log w_{ijt} - \widehat{B}T_{ijt} - \widehat{\beta}_{12}X_{ijt}^2 - \widehat{\beta}_{22}T_{ijt}^2 - \sum_{s=3}^{t} \widehat{\delta}_s = \beta_0 + \beta_{11}X_{0,ij} + \tau_2(t_{0,ij} - 1). \tag{13}$$

This yields a second-stage regression on initial experience and year of job start to obtain $\beta_{11}$ and $\tau_2$.

TABLE VIII

RETURNS TO JOB SENIORITY, TENURE, EXPERIENCE, AND FIRM SIZE, BASED ON VARIOUS REMAINING JOB DURATIONS IN FIRST-STEP MODEL (TABLE 4 OF TOPEL (1991), PANEL B)[a]

| | Remaining Job Durations in Estimating Wage Growth | | | | | |
|---|---|---|---|---|---|---|
| | $\geq 0$ | $\geq 1$ | $\geq 3$ | $\geq 5$ | $\geq 7$ | $\geq 10$ |
| *Denmark* | | | | | | |
| $X_{ijt}$ | 0.0478 | 0.0413 | 0.0341 | 0.0321 | 0.0332 | 0.0342 |
| | (0.0004) | (0.0005) | (0.0006) | (0.0008) | (0.0010) | (0.0015) |
| $T_{ijt}$ | −0.0082 | −0.0067 | −0.0027 | −0.0054 | −0.0058 | −0.0124 |
| | (0.0009) | (0.0010) | (0.0013) | (0.0016) | (0.0021) | (0.0030) |
| $r_{ijt}$ | 0.0079 | 0.0063 | 0.0027 | 0.0032 | 0.0019 | 0.0071 |
| | (0.0003) | (0.0004) | (0.0005) | (0.0006) | (0.0008) | (0.0011) |
| $n_{ijt}$ | 0.0124 | 0.0181 | 0.0205 | 0.0231 | 0.0245 | 0.0227 |
| | (0.0003) | (0.0004) | (0.0005) | (0.0006) | (0.0008) | (0.0010) |
| *Portugal* | | | | | | |
| $X_{ijt}$ | 0.0690 | 0.0732 | 0.0767 | 0.0825 | 0.0819 | 0.0740 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0001) | (0.0001) | (0.0001) |
| $T_{ijt}$ | 0.0154 | 0.0146 | 0.0117 | 0.0092 | 0.0098 | 0.0115 |
| | (0.0014) | (0.0016) | (0.0023) | (0.0033) | (0.0049) | (0.0078) |
| $r_{ijt}$ | 0.0150 | 0.0190 | 0.0221 | 0.0273 | 0.0262 | 0.0278 |
| | (0.0004) | (0.0006) | (0.0008) | (0.0012) | (0.0016) | (0.0024) |
| $n_{ijt}$ | 0.0144 | 0.0126 | 0.0112 | 0.0108 | 0.0125 | 0.0145 |
| | (0.0004) | (0.0006) | (0.0009) | (0.0012) | (0.0016) | (0.0024) |

[a]These are the same regressions as the Topel regressions with seniority index included from our main text (see corresponding column in Table III in the paper), but on various remaining job durations, replicating Topel's (1991) Table 4, Panel B. Standard errors in parentheses.

## B.2. *Topel With Spell Fixed Effects*

Consider now a model in which we allow for heterogeneity in the linear returns to tenure:

$$\log w_{ijt} = \beta_0 + \beta_{11} X_{ijt} + \beta_{12} X_{ijt}^2 + \beta_{21,ij} T_{ijt} + \beta_{22} T_{ijt}^2 + \varepsilon_{ijt}, \tag{14}$$

where we again omit higher-order terms for convenience. Note that this model is somewhat more restrictive than the model in the main text since it does not include individual heterogeneity in linear experience. Taking within-spell first-differences results in

$$\Delta \log w_{ijt} = \beta_{11} + \beta_{21,ij} + \tau_2 + \beta_{12} \Delta X_{ijt}^2 + \beta_{22} \Delta T_{ijt}^2 + \delta_t + \Delta \nu_{ijt}, \tag{15}$$

where the definition of $\delta$ is the same as in Section B.1. Since we have that $\Delta X_{ijt}^2 = 2X_{ijt} - 1$ and $\Delta T_{ijt}^2 = 2T_{ijt} - 1$, this empirical model is also statistically equivalent to that of Altonji and Shakotko (1987), using within-spell first-differences instead of the log wage level, and with $\beta_{21,ij} = \beta_2 + u_{\beta_2,ij}$, where $u_{\beta_i j}$ is a fixed job-specific term. Hence, we can use Altonji and Shakotko's IV method to estimate equation (15), using $\widetilde{\Delta T_{ijt}^2} = 2\widetilde{T_{ijt}} = 2(T_{ijt} - \overline{T}_{ij})$ as an instrument for $\Delta T_{ijt}^2$. As discussed in Topel (1991, p. 167), this method yields equivalent results to taking deviations from the mean combined with a second-step estimator to estimate $\beta_{12}$ and $\beta_{22}$. Such a method yields estimates of $B_{ij} = \beta_{11} + \beta_{21,ij} + \tau_2$, $\beta_{12}$, and $\beta_{22}$, as well as of the dummy variables $\delta_t$. Using the same steps as in Section B.1 to obtain (13), we obtain

$$\log w_{ijt} - \widehat{B}_{ij} T_{ijt} - \widehat{\beta}_{12} X_{ijt}^2 - \widehat{\beta}_{22} T_{ijt}^2 - \sum_{s=3}^{t} \widehat{\delta}_s = \beta_0 + \beta_{11} X_{0,ij} + \tau_2 (t_{0,ij} - 1). \tag{16}$$

From this, $\beta_{11}$ and $\tau_2$ can be estimated. The estimate of $\beta_{21,ij}$ for every $ij$ follows directly. It implies that we are able to identify the distribution of $\beta_{21,ij}$. We focus on the mean in the main text of our paper, which can be consistently estimated by the sample average.

In order to calculate the standard errors, we can use the results of two-step estimators, taking into account that the first- and second-step error terms are correlated. More details are provided in, for example, Murphy and Topel (1985, p. 94, equation (24)) or Wooldridge (2002).

Assume now that instead of equation (14), we have a model that is equivalent with the model of our main text, hence allowing also for heterogeneous effects in linear experience:

$$\log w_{ijt} = \beta_0 + \beta_{11,i} X_{ijt} + \beta_{12} X_{ijt}^2 + \beta_{21,ij} T_{ijt} + \beta_{22} T_{ijt}^2 + \varepsilon_{ijt}.$$

Note that taking within-spell first-differences yields a model that is statistically equivalent to the model in (15). The only difference is the interpretation of the predicted fixed effects that now equal $B_{ij} = \beta_{11,i} + \beta_{21,ij} + \tau_2$, where both $\beta_{11,i}$ and $\beta_{21,ij}$ include random components. Using the same technique as that used to obtain (16) results in

$$\begin{aligned} \log w_{ijt} - \widehat{B}_{ij} T_{ijt} - \widehat{\beta}_{12} X_{ijt}^2 - \widehat{\beta}_{22} T_{ijt}^2 - \sum_{s=3}^{t} \widehat{\delta}_s \\ = \beta_0 + \beta_{11,i} X_{0,ij} + \tau_2 (t_{0,ij} - 1). \end{aligned} \tag{17}$$

This equation is not feasible to estimate, since $\beta_{11,i}$ implies a dummy vector of a dimension that is as large as the number of individuals in the data set. In our paper (see Table III, and the returns to cumulated tenure in Table IV, both under the columns corresponding to the specification for Topel with spell fixed effects), we estimate, therefore, linear tenure and experience effects under the assumption of homogeneous linear returns to experience, as illustrated above in equation (14). Of course, for identifying and estimating the object of our main interest in the paper, the return to seniority, we do not need any extra assumptions—see the main text, equation (10).

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