Wages, Seniority and Separation Rates in a Stochastic Productivity Model: A Comparative Perspective[1]

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Abstract

We discuss a theoretical framework for the job duration of individual workers and the evolution of the wage rate during that job assuming that individual productivity follows a geometric Brownian. A comparative overview of job search, random learning and random growth models is put forward as background to the literature on job tenure distribution. The random growth model fits best the hump-shaped tenure profile observed in data on job separation rates and is consistent with empirical evidence that log wages follow a random walk. We provide a synopsis of the persisting debate on the returns to job seniority adding a non-deterministic tenure profile perspective. The specification of the model allows the application of option theory to calculate the value of a job and the optimal job separation rule. An extension to the initial model adding log firm size is introduced.

1 Introduction

This paper targets the ever recurring themes of the job tenure distribution, wage rate evolution and the relationship between job seniority and individual wages. Though a lot of work has been undertaken in this field, the mission has not been accomplished yet: the factors triggering job separation remain still unclear to our day while the debate on whether we do or we do not have tenure profiles on wages seems to continue relentlessly. We undertake a relatively new approach assuming a model with an unpredictable evolution of the productivity match after the start of a job, rooted in the random growth productivity framework developed in Teulings and Van der Ende (2000). Starting a job demands some specific investments; when the match productivity does not evolve favorably, the investments lose their value and separation becomes the only efficient alternative.

Most theories on the determinants of the distribution of job tenures have focused on search and learning models. In search models workers keep their present job until they find a better one. The chances of finding an even better job diminish as the selection process proceeds. Hence in the search framework job duration increases with the worker's labour market experience, e.g. Jovanovic (1979b), Burdett and Mortensen (1998). In random learning models a worker and a firm start a job without knowing the quality of their match, this quality being revealed in time. The productivity has a constant match-specific mean. Workers periodically observe productivity and quit to another job whenever their mean productivity is below a standard that increases with tenure. This process continues until the workers find jobs in which their inferred productivity will be above-standard, e.g. Jovanovic (1979a), Miller (1984).

Far less attention has been paid to models with a random evolution of job productivity. Such a model assumes a stochastic evolution of match productivity after the date of the job start. In particular, individual productivity is assumed to follow a geometric Brownian motion (a continuous-time random walk in logarithms). Specific investments such as hiring costs or firm-specific training costs are required at job start. When the match productivity falls below a threshold, the worker and the firm separate. The model has strong predictive accuracy in underlying empirical data. Teulings and Van der Ende (2000) have shown in this sense that the random growth productivity model can explain data observations on tenure distribution better than search or random learning models. Furthermore, by immediate implication of the model, the logarithm of the firm size and the logarithm of the wages would also follow a random walk. Both these findings are supported empirically: first, the firm size evolves approximately according to Gibrat's law: Jovanovic (1982) provides well-documented evidence that the Gibrat law tends to hold for large firms; second, several papers strongly support the fact that log wages approximately follow a random walk, e.g. Abowd and Card (1989) or Topel and Ward (1992). Given all the evidence on tenure distributions, evolution of firm size and wages, we find surprising that the random walk assumption has not been applied before in this context. A model of random proportional growth at firm-level employment scale has been derived by Bentolila and Bertola (1990). The close relationship between this model and the random productivity model at individual level has been revealed in Teulings and van der Ende (2000). In Bentolila and Bertola's model a firm is increasingly uncertain about the productivity of its employees in a more distant future and any random shocks to future productivity are in fact shocks in the firm's demand curve. We will revisit in this paper Teulings and van der Ende's model next to submitting an extension of the original set-up to include firm size. The generalized model can account in an elegant way for implications of Kuhn's (1989) last-in-first-out (LIFO) layoff rule and it can provide a start for further research on insider-outsider theories of the labour market.

Teulings and Van der Ende used the random growth model to explore *inter alia* the relationship between job seniority and wages. There is by now a considerably large literature on job tenure profiles in wages: e.g. Altonji and Shakotko (1987), Abraham and Farber (1987), Topel (1991), Altonji and Williams (1997), Farber (1999).

The conclusions are however diverging. Some authors have suggested that tenure profiles might be fully explained by selection bias, since good jobs and good workers survive. The consequence would then be that the high wage of high tenure workers is merely an artefact, not being due to job seniority but rather to favorable characteristics of the respective workers. Other authors claim that wage-tenure profiles are empirically important and that the wage increases due to additional years of tenure are large. We shall therefore include in this paper a synopsis of the debate on the returns to job seniority. Given the high interest on the subject in existing literature, it is striking that no one has explicitly considered the possibility of a non-deterministic tenure profile, as implied by the random growth productivity model. The survival of a selective sample of random walks generates a tenure profile that is partially consistent with Topel (1991) and that at the same time puts under fire much of the previous work.

The methodology of the model discussed in this paper is based on the application of option theory. We mentioned already that specific investments when the job starts are required; this leads to irreversible hiring and separation decisions. By analogy one can attach option values to these investments. Given the geometric Brownian motion used as underlying productivity path, we are able to apply Dixit's (1989) option theory to calculate the value of a job and to derive optimal job separation rules in a similar way as within the theory of financial options. In particular, the use of option theory provides a powerful theoretical apparatus for the analysis of the firm's optimal strategy, similar to Bentolila and Bertola's (1990) framework. The model can be enabled, as an extension, to test and treat implications of hold up problems and insurance issues, given that Dixit's theory can be immediately applied in a non-risk-neutral environment.

We will structure the paper as follows: a background discussion on several theories of tenure distribution and job exit rates will be the subject of the 2^{nd} section; section 3. will present empirical evidence on wage rate progress and will overview a much controversial debate over the wage returns to seniority; in section 4. we will highlight the specifications of the random growth model, extend these initial specifications to include firm size as well and discuss comparatively links with existing literature; finally a summary of this study and further research avenues are put forward in the last section.

2 Models of tenure distribution and optimal separation

2.1 Search models

The bulk of studies on labour market dynamics made extensive use of search or learning models. On a closer scrutiny the search framework seems to have actually been the long-time favoured one. In a typical search environment a worker faces an individual labor market, as reviewed in Lippman and McCall (1976). In his labour market a worker without relevant additional outside options, for instance a male worker in the working age range, may at any moment be offered a job. Ignoring non-pecuniary gains, the duration of a job is typically related to the job offer arrival rate and a wage distribution. Therefore in search models we have two types of stochastic shocks that might influence the separation decision. The first shock is the arrival process of new employment offers and the second is, conditional on the arrival, the value of those offers. We have separation when the value of a recent offer exceeds the value of the current job. Both shocks are modeled as transitory shocks in the job search literature. This means that the probabilities of job offer arrivals are not correlated over time. A major implication of the job-search theory is that the number of offers received by a worker increases with the time he spends on the labor market. If however the best of all these offers is his current job, then in expectation the maximum will increase with experience. Hence the probability of receiving an even better job and thus of separating declines with experience; this generally underlines the empirical findings in the relative long-run, such as for instance quarterly or annual basis. Nonetheless empirical research shows that the probability of leaving a job is actually increasing in the first several months of employment and decreasing thereafter. Or standard search models do not account for this initial increase in exit rates with tenure. A more detailed structural model is required to explain this phenomenon; in particular learning and random growth models have been proved to better fit the empirical data, especially on the short term.

We consider one of the standard search models in the literature, Jovanovic (1979b), and discuss its main assumptions. The model by Jovanovic (1979b) is a model of permanent separations, which the author includes under the category of "pure-search-goods" models of job change. Similar types of models have been previously discussed in Burdett (1978) or Mortensen (1978). Next to on-the-job search intensity Jovanovic considers firm-specific human capital investment, his paper focusing on the relationship between firm-specific human capital and the likelihood of future job separations. The worker's search intensity determines the arrival rate of new wage offers. These new offers are drawn independently from the wage-offer distribution characterized by the cumulative distribution function $F(w)$. We denote by $\lambda(t)\Delta t + o(\Delta t)$ the probability that a wage offer will arrive during the time interval $(t, t + \Delta t)$. Conditional on the distribution $F(w)$, the worker's optimal policy is characterized by a reservation wage $\theta(t)$. The job ends therefore as soon as a wage offer exceeding $\theta(t)$ is received. If we further define the survivor function as

$$\overline{F}(w) \equiv 1 - F(w) \tag{1}$$

and

$$h(t) \equiv \lambda(t)\overline{F}[\theta(t)] \tag{2}$$

then we will have $h(t)\Delta t + o(\Delta t)$ as the probability that an acceptable offer arrives on the interval $(t, t + \Delta t)$. Assume that the fraction $s(t)$ of the worker's time is devoted to on-the-job search while another fraction, $\phi(t)$, is devoted to on-the-job training, with $s(t) + \phi(t) \in [0, 1]$. Denote by $x(t)$ the worker's productivity on a particular job, where one can write

$$x(t) = \mu + k(t) \tag{3}$$

μ is the quality of the employer-worker match possessing a distribution $F(\mu)$ across matches and $k(t)$ is the human capital stock accumulated through training on the job. Jovanovic makes the crucial assumption that the productivity $x(t)$ evolves according to the following law

$$\frac{dx(t)}{dt} = g[\phi(t)x(t)] - \delta x(t),\; x(0) = \mu \tag{4}$$

where $g(0) = 0$, $g'(.) > 0$, and $g''(.) < 0$. This equation states that at $t = 0$, the productivity of the worker is equal to μ; afterwards productivity can be increased by doing on-the-job training. If no time is devoted to this investment (if $\phi(t) = 0$), productivity depreciates at a rate δ. The worker's wage is assumed to be equal to his net marginal product, where the actual amount produced by the worker is proportional to the fraction of time $(1 - \phi - s)$ that he decides to spend working. Hence,

$$w(t) = [1 - \phi(t) - s(t)]x(t) \tag{5}$$

As one can notice, Jovanovic does assume that all training and search costs are to be paid by the worker; the worker also gets all the rents associated with being well matched and those associated with a particular human-capital stock and this while other previous models yield the conclusions that it would be optimal for the rents associated with a good match to be shared between the worker and the employer, e.g. Mortensen (1978). This apparently unique rent sharing assumption is nevertheless not essential in the light of Jovanovic's (1979b) main rationale; he argues that even if the assumption that all the rents go to the worker were totally unacceptable, the results of his paper would still be relevant since they do in fact characterize that particular turnover, job-search, and respectively human capital-investment behavior that will maximize each worker's lifetime discounted expected marginal product[1].

Jovanovic's model also postulates that the accumulated on-the-job training is completely firm specific, general human capital being ignored here for the sake of simplicity. The separation condition is inferred from equations (3) and (4) above:

$$\mu' > \mu + k(t) \tag{6}$$

where μ' is the quality of the match with a potential new employer, while μ and $k(t)$ are the current match quality and respectively current job accumulated human-capital stock. The change in the productivity evolution equation (4) is that the initial condition becomes $x(0) = \mu'$. Jovanovic further defines the following: $V[x(t), t]$ as the value to the worker of having a productivity equal to $x(t)$ at t, with $0 \leq t \leq T$; $R(t)$ as the probability that the current job episode will end before calendar time t. Having $h(t)$ defined in (2), we can write

$$R(t) = 1 - e^{-\int_{t_0}^{t} h(y)dy} \tag{7}$$

The wage offer arrival rate is by hypothesis increasing and concave in the fraction of time spent searching, $s(t)$: $\lambda = \lambda[(s(t)]$, $\lambda(0) = 0$, $\lambda' > 0$, $\lambda'' < 0$. By setting marginal cost of search equal to marginal cost of return and performing a few derivations, Jovanovic obtains the following:

$$x(\tau) = \lambda'[s(\tau)] \int_{x(\tau)}^{\infty} \{V(y, \tau) - V[x(\tau), \tau]\} f(y)dy \tag{8}$$

In order to interpret expression (8) one can differentiate totally with respect to $x(\tau)$ while holding τ constant:

[1] Presumably there are many different sharing arrangements that lead to exactly the behaviour in this paper; Jovanovic mentions Mortensen (1978) as having addressed this issue.

$$D \equiv \left.\frac{ds\,(\tau)}{dx(\tau)}\right|_{\tau \text{ constant}} = \frac{\lambda'[s(\tau)]}{x(\tau)\lambda''[s(\tau)]}\left\{1 + \lambda'[s(\tau)]V_x[x(\tau),\tau]\overline{F}[x(\tau)]\right\} \tag{9}$$

Since we had $\lambda' > 0$ and $\lambda'' < 0$ by hypothesis, we obtain from (9) that

$$D \equiv \left.\frac{ds\,(t)}{dx(t)}\right|_{t \text{ const}} < 0 \tag{10}$$

It appears thus that the amount of time devoted to search for alternative employment, $s(t)$, decreases with $x(t)$ holding t fixed. Jovanovic's conclusion is therefore that those who are better matched and those that have more specific human capital spend less time searching. He also verifies that separation probabilities as a function of tenure are uniformly lower for those who are well matched for two reasons: firstly, from (10), those workers who are well matched spend less time searching for alternative work, and secondly, when they do receive alternative offers, they are less likely to accept them. Certainly one main problem with this conclusion and with the model assumptions is that "being well matched" or "being badly matched" is fixed; in other words the employer-worker match value is exogenously set, both parties knowing it with certainty since the moment they start their employment relationship. As formalized in (4), individual productivity can only change with the amount of the job training undertaken (depreciating at a given rate if there is no investment in this sense) or if the worker quits to another job characterized by a better "match parameter". Or this setting is too rigid to describe a dynamic labour market where there is uncertainty about the future productivity. The problem is in fact common to conventional job search models.

We have thus seen that although Jovanovic's (1979b) is a " classical" in terms of job search models, it does gain mathematical elegance and ease in interpretation at the expense of a rigid assumptions set. We will shortly characterize a more recent search model, different in certain aspects from the model in Jovanovic, the wage posting game in Burdett and Mortensen (1998). Let us consider a continuum of homogeneous employers that choose permanent wage offers and a continuum of homogeneous workers[2] that search by randomly and sequentially sampling from this set of offers. The measure of workers is m,while the measure of employers is normalized to 1. Burdett and Mortensen model the unemployment alternative, having that at any moment in time each worker is either in state 0 (unemployed) or in state 1 (employed). At random time intervals a worker receives information about a new or alternative job opening. The rate of arrival is characterized by a Poisson process and it depends on the worker's current state; λ_i is the parameter of the Poisson arrival process with $i \in \{0, 1\}$. An offer is assumed to be the realization of a random draw from F, the distribution of wages among employers. As in Jovanovic, workers must respond to offers as they arrive and there is no recall. Workers move from lower to higher paying jobs when opportunity arises (jobs are identical apart from the associated wage) but they also move from employment to unemployment and viceversa. A particular assumption of the model is made for the rate of separation: job-worker matches are destroyed at an exogenous positive rate δ. Furthermore, any unemployed worker receives flow benefits b per instant. All agents discount future earnings at rate r. Given this framework, the expected discounted lifetime income of an unemployed worker, V_0, can be expressed as the solution to the following asset pricing equation:

$$rV_0 = b + \lambda_0 \left[\int \max\{V_0, V_1(\widetilde{x})\} dF(\widetilde{x}) - V_0 \right] \tag{11}$$

Equation (11) states that the opportunity cost of searching while unemployed is equal to income while unemployed plus the expected capital gain attributable to searching for an acceptable job, where

[2] The worker heterogeneity case is also tackled in Burdett and Mortensen (1998) but we limit ourselves to their initial model where all workers and all firms are respectively identical. For our purpose relaxing the homogeneity assumption is not essential.

acceptance occurs only if the value of employment, $V_1(x)$, exceeds that of continued search. In a similar way one can obtain that the expected lifetime income of a worker currently employed at wage rate w is the current income plus the expected gain from searching for a better job minus the loss in the eventuality of getting unemployed:

$$rV_1(w) = w+\lambda_1 \int [\max\{V_1(w), V_1(\widetilde{x})\}-V_1(w)]dF(\widetilde{x})+\delta[V_0-V_1(w)] \tag{12}$$

A reservation wage R is further introduced such that

$$V_1(w) \gtreqless V_0 \text{ as } w \gtreqless R \text{ where } V_1(R) = V_0 \tag{13}$$

Using the results in expressions (11), (12) and (13), Burdett and Mortensen (1998) are able to derive

$$R - b = [\lambda_0 - \lambda_1] \int_R^{\infty} \left[\frac{1 - F(x)}{r + \delta + \lambda_1(1 - F(x))} \right] dx \tag{14}$$

Letting the ratio of the discount factor to the arrival rates of jobs for the unemployed pool tending to zero, $r/\lambda_0 \to 0$, the expression above can be simplified to

$$R - b = [k_0 - k_1] \int_R^{\infty} \left[\frac{1 - F(x)}{1 + k_1[1 - F(x)]} \right] dx \tag{15}$$

where $k_0 = \lambda_0/\delta$ and $k_1 = \lambda_1/\delta$ represent the ratios of state-dependent arrival rates to the job separation rate.

Given a reservation wage R, the flow of workers in and out of unemployment is straightforward. In the steady state, the flow into employment equals the flow from employment to unemployment,

$$\lambda_0[1 - F(R)]u = \delta(m - u) \tag{16}$$

Using equation (16) one can solve for the equilibrium unemployment rate:

$$u = \frac{m}{1 + \frac{\lambda_0}{\delta}[1 - F(R)]} \tag{17}$$

Burdett and Mortensen (1998) are further using the number of employed workers receiving a wage no greater than w at time t, i.e. $G(w,t)(m-u(t))$, where $G(w,t)$ is the proportion of employed workers at t receiving a wage no greater than w and $u(t)$ is the measure of unemployed at t, and compute its time derivative. This is written as the difference between the unemployed workers' flow into the labour market for wages no greater than w and the flow into unemployment or respectively into higher paying jobs:

$$\begin{aligned}\frac{dG(w,t)(m-u(t))}{dt} &= \lambda_0 \max\{F(w)-F(R),0\}u(t) \\ &\quad -[\delta+\lambda_1(1-F(w))]G(w,t)(m-u(t)) \end{aligned} \tag{18}$$

Then for all $w \geqslant R$,using (17) and the above, one can write down the unique steady-state distribution of wages earned by employed workers:

$$G(w) = \frac{F(w)-F(R)}{[1+\frac{\lambda_1}{\delta}(1-F(w))][1-F(R)]} \tag{19}$$

Focusing on the steady-state, the number of workers earning a wage in the interval $[w-\varepsilon, w]$ is represented by $[G(w)-G(w-\varepsilon)](1-u)$, while $F(w)-F(w-\varepsilon)$ is the measure of firms offering a wage in the same interval, where ε is an arbitrarily small positive quantity. Then the measure of workers per firm earning a wage w is given by:

$$l(w|R,F) = \lim_{\varepsilon \to 0} \frac{G(w)-G(w-\varepsilon)}{F(w)-F(w-\varepsilon)}(m-u) \tag{20}$$

Further assuming that $l(w|R,F) = 0$ if $w < R$ and writing $F(w) = F(w^-)+\nu(w)$ where $\nu(w)$ is the mass of firms offering wage w, we get:

$$l(w|R,F) = \frac{mk_0 \frac{1+k_1(1-F(R))}{1+k_0(1-F(R))}}{[1+k_1(1-F(w))][1+k_1(1-F(w^-))]}, \text{ for } w \geqslant R \tag{21}$$

On the firm side, conditional on R and F, each employer is assumed to post a wage that maximizes its steady-state profit flow:

$$\pi = \max_{w}(p - w)l(w|R, F) \tag{22}$$

At this point we can characterize the equilibrium to the search and wage-posting game that was discussed above. Burdett and Mortensen state that the equilibrium solution is a triple (R, F, π) such that R satisfies (15), π satisfies (22) and F satisfies the following

$$\begin{gathered}(p - w)l(w|R, F) = \pi, \ \forall w \text{ on support of } F \\ (p - w)l(w|R, F) \leq \pi, \text{ otherwise}\end{gathered} \tag{23}$$

Two restrictive assumptions need to be made. First, to rule out the trivial, the productivity of workers is greater than the common opportunity cost of employment: $\infty < p > b \geqslant 0$. Second, we assume that the ratios of state-dependent arrival rates to the job separation rate are finite strictly positive: $k_i \in (0, \infty)$. This second assumptions excludes the limiting cases of the competive and respectively the monopsony solutions.

Having proved that non-continuous wage offer distributions are excluded from the solutions set (see Burdett and Mortensen (1998) for the detailed proof) simplifies considerably the initial expression (21):

$$l(\underline{w}|R, F) = \frac{mk_0}{(1 + k_0)(1 + k_1)} \tag{24}$$

where $\underline{w}$ denotes the infimum of the support of the equilibrium F. (24) tells us that the number of workers available to a firm offering a particular wage and conditional on wages offered by other firms is not depending on the wage offered as long as $\underline{w} \geq R$. This further implies that the employer offering the lowest wage in the market will maximize its profit flow iff $\underline{w} = R$.

Based on the assumptions above Burdett and Mortensen are able to derive successively:

$$\pi = (p - w)l(w|R, F) \ \forall w \text{ in the support of } F \tag{25}$$

$$F(w) = \left[\frac{1 + k_1}{k_1}\right]\left[1 - \left(\frac{p - w}{p - R}\right)^{\frac{1}{2}}\right] \tag{26}$$

and using $F(\overline{w}) = 1$, where $\overline{w}$ is the supremum of the support of the equilibrium F,

$$p - \overline{w} = \frac{p - R}{(1 + k_1)^2} \tag{27}$$

Solving for R by using (26) and (27), one obtains:

$$R = \frac{(1 + k_1)^2 b + (k_0 - k_1)k_1 p}{(1 + k_1)^2 + (k_0 - k_1)k_1} \tag{28}$$

Equations (25) to (28) completely characterize the unique equilibrium, once Burdett and Mortensen have proved that no wage off the support of the candidate F yields higher profits. Interesting from this result is the implied positive relationship between the wage offer and the employer labor force size. This anticipates one of the research ideas we will expand on in the last section of this paper, the link between a worker's career and earnings prospects and the firm size. Burdett and Mortensen conclude in this respect that as the voluntary quit rate $\lambda F(w)$ decreases with the wage offer, larger firms experience lower quit rates. And because workers only switch employers in response to a higher wage offer, workers with either more experience or tenure are more likely to be earning a higher wage.

2.2 Random learning models

In learning models the key feature is that workers and firms have no apriori information about the quality of their match. In each period the match produces a stochastic output with a constant match-specific mean. The worker and the employer gradually learn about match quality by observing the realizations of this random output. The productivity match level can therefore be learned with increasing certainty implying that any shocks will have a permanent effect on the probability of future separations. This is in clear contrast to search models where the shocks, as seen from the earlier presentation, are transitory. Even though the effects of the shocks are permanent, the share of each shock will be however decreasing the more shocks are accumulated. Learning models generate the hump-shaped separation rate observed in practice, increasing in the short term, but decreasing after a certain period[3], which search models overlook. However, as Teulings and Van der Ende (2000) demonstrate, the observed separation rates do not decline as quickly as predicted by the estimated learning model in either Lancaster, Imbens, and Dolton (1987) or Miller (1984), the random growth model based on the assumption that productivity follows a geometric Brownian better fitting the empirical data from this point of view.

One of the earliest random learning models has been developed in Jovanovic (1979a). It is assumed therein that the match-specific productivity per period is constant throughout a job and that its level is drawn from a probability distribution identical for, and known by, all workers and firms. The match-specific probability is inferred from the productivity observed after the start of a match. Since this productivity is observed with error, the perception of productivity changes over time. What remains the same however is the perception of productivity in every new match, with a new worker to the firm and a new firm to the worker. The model further supposes an infinite worker lifetime and no retirement age. On the firm side, there are constant returns to scale with labor being the only factor of production. Finally, Jovanovic's model does not bear any informational asymmetries, each worker's output being assumed to be observed instantaneously by the worker and by the employer. In such a set-up we will obviously have that all turnover is generated by

[3] Van der Ende (1997) states that the peak of the observed job exit rates is reached at about three months after the start of a job.

revealing the information about the match quality, which simplifies things a lot. Nonetheless it is interesting to overview the model's predictions about the rates of job change. Given the assumptions above, the contribution of one worker to the total output of the firm over a period of length t is given by:

$$X(t) = \mu t + \sigma z(t) \tag{29}$$

where μ and $\sigma > 0$ are constant and $z(t) \sim N(0, t)$, implying $X(t) \sim N(\mu t, \sigma^2 t)$. Whereas σ is the same for each worker-firm match, μ differs across matches; μ is the quality measure of the match. When the match is formed μ is unknown, more information about it being acquired as the match evolves. Jovanovic assumes this quality measure to be normally distributed with a mean m and a variance s, $\mu \sim N(m, s)$, with a job change simply meaning a draw of a new value of μ from this normal distribution, with independent successive draws. In this setting, firms compete for workers by offering wage contracts. The wage policy of a firm can be characterized by a function $w(X(t), t)$. All job separations are at the worker's initiative, so they are modeled as quits in Jovanovic's model. We can then write the value of quitting a job as

$$Q = \alpha(Q, w(X(t), t)) - c \tag{30}$$

where $\alpha(Q, w(.))$ is the present value to the worker of obtaining a job in a firm that offers a wage function $w(.)$ as the wage contract; c is the direct and forgone earnings costs of job changing. If further we denote by T the quitting time, by F the probability that the worker quits before tenure t and by H the probability that the worker does not quit before tenure t and that by that time his cumulative output does not exceed x, then the appropriate densities are

$$\begin{aligned} h(x, t|w(.), Q) &= \frac{\partial H}{\partial x} \\ f(t|w(.), Q) &= \frac{\partial F}{\partial t} \end{aligned} \tag{31}$$

At the optimally chosen h and f we will then have

$$\alpha(Q, w(.)) = \int_0^\infty e^{-rt} \left(\int_{-\infty}^\infty whdx \right) dt + Q \int_0^\infty e^{-rt} f dt \tag{32}$$

The discounted revenue from the output of a single worker is obtain by differentiating equation (29),

$$E\int_0^\infty e^{-rt}dX(t) = E\int_0^\infty e^{-rt}E_{X(t)t}(\mu)dt + E\int_0^\infty e^{-rt}\sigma E_{X(t)t}dz(t) \tag{33}$$

The stochastic integrals in (33) above are Ito integrals and the last integral in the right-hand side is therefore 0, by the independent increments property of the Wiener process $z(t)$. Given this, Jovanovic defines the following

$$\begin{aligned}\beta(Q, w(.)) &\equiv E\int_0^\infty e^{-rt}E_{X(t)t}(\mu)dt \\ &= \int_0^\infty e^{-rt}\left(\int_0^\infty E_{xt}(\mu)h(x,t|Q,w(.))dx\right)dt \end{aligned} \tag{34}$$

Using $\beta(.)$, the discounted expected net revenue from the employment of a certain worker having a wage contract $w(.)$ and a present value of quitting Q is

$$\pi(Q, w(.)) = \beta(Q, w(.)) - \alpha(Q, w\,(.)) + \gamma(Q, w(.)) \tag{35}$$

where

$$\gamma(Q, w(.)) = Q\int_0^\infty e^{-rt}f(t|Q,w(.))dt \tag{36}$$

The equilibrium conditions are obtained as follows. Denote by B the set of competitive equilibrium wage contracts, and for any $w(.)$ let $Q(w(.))$ be the unique solution for Q from equation (30). Then, if $w(.) \in B$, the following are true: each worker follows his optimal quitting policy in response to $w(.)$ and to $Q(w(.))$; $\pi\{Q, w\} \geq \pi\{Q, \widehat{w}\}$ for all $\widehat{w}(.) \neq w(.)$, so that $w(.)$ maximizes expected profits; and $\pi\{Q(w(.)), w(.)\} = 0$ as the zero expected profit constraint. Given these conditions above, the equilibrium wage contract will consist in the worker being paid his expected marginal product at each moment in time, while the equilibrium quitting value being the quitting value associated to this equilibrium wage function:

$$w^*(x,t) = E_{xt}(\mu)\forall(x,t) \tag{37}$$

$$Q^* = Q(w^*(.)) \tag{38}$$

What interests us mostly in Jovanovic's (1979a) model is his approach to determining the tenure distribution and job separation probability. Considering the result in (37) and the fact that $w(t)$ is normally distributed with mean s and variance $S(t)$ for all t, one can write the stochastic differential equation:

$$dw(t) = S(t)\sigma^{-1}dz(t),\ w(0) = m \tag{39}$$

If $V(w,t)$ labels the current value of the game to the worker with tenure t and wage $w(t) = w$, then

$$V(w,t) = w\Delta t + e^{-r\Delta t}E_{wt}V(w[t+\Delta t],t) + o(\Delta t) \tag{40}$$

Applying Ito's Lemma and taking the limit $\Delta t \to 0$ in (40), we get

$$w - rV(w,t) + \frac{S(t)^2}{2\sigma^2}V_{ww}(w,t) + V_t(w,t) = 0 \tag{41}$$

Along the boundary $[\theta(t),t]$ with $V[\theta(t),t] = Q$ where $\theta(t)$ is the reservation wage at which the worker quits the firm, we have by the optimal stopping condition theorem that

$$V_t[\theta(t),t] = \frac{\partial Q}{\partial t} = 0 \tag{42}$$

which implies

$$\theta(t) = rQ - \frac{S(t)^2}{2\sigma^2}V_{ww}[\theta(t),t] \tag{43}$$

As one can notice from expression (43), the limit of the reservation wage is rQ. This happens because the wage tends to a constant as t tends to infinity. In order to approximate the job separation probability, Jovanovic indeed sets $\theta(t) = rQ$. Then:

$$F(t|w(.), Q) = 2\left[1 - N\frac{m - rQ}{p(t)^{\frac{1}{2}}}\right] \tag{44}$$

with density

$$f(t|w(.), Q) = \frac{(2\pi)^{-\frac{1}{2}}(m - rQ)}{p(t)^{\frac{3}{2}}} e^{-\frac{(m-rQ)^2}{2p(t)}} \tag{45}$$

where

$$N(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{x} e^{\frac{z^2}{2}} dz \tag{46}$$

and $p(t) = s - S(t)$ is the precision. Define the hazard rate as a function of the separation probability and the separation probability density

$$\phi(t) \equiv \frac{f}{1 - F} \tag{47}$$

Expression (47) is in fact the density of separation conditional upon attainment of tenure t.

The model predicts a non-monotonic relationship: in the beginning $\phi'(t) > 0$, while $\phi'(t) < 0$ as t gets relatively large (which explains the behavior observed in empirical data, as discussed in the introductory section)[4]. Using the previously computed expressions, the tenure-wage profile is given by:

$$\widehat{w}(t) = m + (m - rQ)\frac{2N\left\{-a[s - S(t)]^{-\frac{1}{2}}\right\}}{1 - 2N\left\{-a[s - S(t)]^{-\frac{1}{2}}\right\}} \tag{48}$$

From the last equation, $\widehat{w}(t)$ increases monotonically from m to $m + (m - rQ)\frac{2N\left(-as^{-\frac{1}{2}}\right)}{1-2N\left(-as^{-\frac{1}{2}}\right)}$. One of the implications of the model is

[4] For a complete rationale behind this conclusion, see Jovanovic (1979a), page 981.

that the average wage of a cohort of workers increases with tenure, eventually at a decreasing rate, as low-wage workers quit and high-wage workers stay. Moreover, as tenure becomes indefinitely large, the average wage of those members of the cohort who have not quit approaches a constant as the wage of each worker becomes constant and equal to his true productivity. A second prediction of Jovanovic's model is that a mismatch leads to a lower wage and thus to an earlier separation. In this respect, holding constant market experience, average past earnings are likely to be lower for a worker who has experienced many job separations.

A couple of subsequent models have criticized the pure random learning specification in Jovanovic (1979a). Lancaster, Imbens and Dolton (1987), for instance, indicate that the original model of Jovanovic is mispecified (see also Van der Ende (1997)) and cannot accurately describe the empirical data. Miller (1984) generalizes the model of Jovanovic (1979a) allowing for different job occupations with type-specific distributions of productivity and observation error. We will briefly discuss some of the main results of this model. The agent's return from working in the mth job at time $t \in T \equiv \{0, 1, 2...\}$, chosen from the job set M, is the sum of three components:

$$x_{mt} = \psi_t + \xi_m + \sigma_m \epsilon_{mt} \tag{49}$$

where ψ_t denotes a time-trended variable independent of the job and observed regardless of whether the individual works (might be due to business cycles, effects of age, or general experience and formal education); ξ_m is a time-invariant match parameter, which the individual does not observe directly but believes that $\xi_m \sim N(\gamma_m, \delta_m^2)$ before acquiring any experience in the job; the third term is never observed but σ_m is known and $\epsilon_{mt} \sim N(0, 1)$. Using this notation, Miller gives a first definition: two jobs m and m' belong to the same occupation n iff $(\gamma_m, \delta_m, \sigma_m) = (\gamma_{m'}, \delta_{m'}, \sigma_{m'})$. Their "common information factor" is defined as

$$\alpha_m \equiv \frac{\sigma_m^2}{\delta_m^2} \tag{50}$$

A job is thus completely characterized by the population mean m and by its common information factor α_m, which is the fraction between the variance of the job and the variance of the measurement error in productivity. What Miller shows is that the jobs with the highest mean productivity and the highest information factors are the most attractive and should consequently be tried first. Miller's conclusion implies that young workers have a comparative advantage in these jobs. This is because jobs with high informational benefits pay less on average in equilibrium and they do attract thus the inexperienced who discover their personal match relatively fast. There is also a lower probability for a young worker that within a particular occupation his match will be superior, while an older worker has had plenty of time to find such a superior match. Miller also inserts an empirical application to his model: he tests the theoretical framework developed in a sample of 1969 tenure data for white American men assuming a one-occupation economy and using level of schooling as control variable. His main outcome is that there must be other reasons than pure random learning that significantly affect job tenures. For instance, he launches the hypothesis that job turnover rates depend on lifetime socioeconomic characteristics, for which employment groups serve as a proxy. Inasmuch as predictive accuracy is concerned, although enhanced from the simple model considered in Jovanovic (1979a), Miller's model also fails to adequately fit the data. His estimation results indicate that the model used is mispecified; inter alia, he observes that the estimated hazards are considerably biased upward.

2.3 Random growth models

The main assumptions of the random growth models can be said to be the mirror image of the ones in the learning models, see Teulings and Van der Ende (2000) for a short introductory comparison. While the worker and the firm are perfectly knowledgeable about the current match productivity, they do not know its future evolution. The key point is that future productivity is by hypothesis following a random walk. In this setting the separation becomes the efficient alternative when the productivity of a match falls below a certain threshold. As in the random learning model, shocks to productivity have permanent effects on future separations. However, unlike in their random learning counterpart, the effect of the new shocks does not decline in time. Accordingly, while in learning models the uncertainty about the productivity is decreasing with tenure, in the random growth models the agents are increasingly uncertain about future income. Workers and firms are required to make specific investments at the start of the match. Types of these investments can be hiring costs, firm specific-training or, more general, any costs associated with the time a worker needs to get acquainted with his new job. One essential property of random growth models is that the specific investments lose their value upon separation being in this sense different from models with temporary layoffs and possibility of recall, as discussed in Feldstein (1976). Below we briefly describe a model of random growth in a framework of labour demand for firms under uncertainty.

Bentolila and Bertola (1990) study a model of firing and hiring of workers with randomly changing profit expectations. Turnover costs are modelled as linear, asymmetric adjustment costs. Keeping the authors' notation, the model setup consists in a firm with linear production technology, using only homogeneous labour, L, and facing a constant elasticity demand function captured by the following two expressions:

$$Q_t = A_t L_t \tag{51}$$

$$Q_t = Z_t P_t^{\frac{1}{\mu - 1}} \tag{52}$$

where Q_t - production and sales at time t; P_t - product price; μ -inverse of the markup factor; A - labour output factor; Z_t - market index capturing an exogenous evolution of demand. The only uncertainty in this model arises from movements in demand, $\{Z_t\}$ being a geometric Brownian with drift:

$$dZ_t = Z_t \vartheta_z dt + Z_t \sigma_z dW_t \tag{53}$$

with ϑ_z the constant mean growth rate, σ_z the standard deviation and $\{W_t\}$a standard Wiener process with independent, normally distributed increments. Although from the equation above demand is growing in expectation at an exponential rate ϑ_z, the actual rate of growth is random, and the further outlook is more and more uncertain: in this regard the implied effect of the random shocks of the market index is that a firm should permanently adjust its income expectations. An observation is that while the model discussed in Bentolila and Bertola (1990) assumes for simplicity a deterministic productivity growth at exponential rate ϑ_a and a constant wage rate, the authors argue that generalization is straightforward in working with stochastic productivity, decreasing returns to scale, stochastic input prices or wages following a geometric Brownian with known parameters.

Bentolila and Bertola's model tests Gibrat's law of proportional growth. The Gibrat law states that growth of log firm size does not depend on firm size. If we condition on the labour factor L_t the Gibrat law holds when the firm size is measured as the firms revenues. Combining expressions (51) and (52) one obtains the firm revenues at time t:

$$P_t Q_t = Z_t^{1-\mu}(A_t L_t)^{\mu} \tag{54}$$

Keeping in mind (53), it is straightforward that holding L_t constant in (54), the log revenues are a Brownian motion with drift, which constitutes the prove for Gibrat's law. As shortly mentioned before, on the empirical side it is well-documented that the Gibrat law tends to hold for large firms, e.g. Jovanovic (1982).

The firm is choosing an employment policy so as to maximize its objective function, the expected present value of cash flows over the infinite future:

$$\begin{aligned} V_t \equiv \max_{\{X_t\}} E_t\{\textstyle\int_t^{\infty} e^{-r(\tau-t)}[(Z_\tau^{1-\mu}(A_\tau L_\tau)^{\mu} - wL_\tau)d\tau \\ -(1_{[dX_t>0]}H - 1_{[dX_\tau<0]}F)dX_\tau]\} \end{aligned} \tag{55}$$

subject to the dynamic accumulation constraint

$$dL_t = dX_t - \delta L_t dt \tag{56}$$

where r - required rate of return; $1_{[.]}$ - indicator function; $\{X_t\}$ - cumulative labor turnover process with $dX_t > 0$ when hiring and $dX_t < 0$ when firing; δ - exogenous worker quit rate. Assuming additionally that hiring and firing costs are constant, that wages are constant and that all workers are homogeneous, Bentolila and Bertola (1990) are able to get the marginal revenue product of labour by computing the partial derivative of the value function V_t with respect to current employment. Defining

$$\eta_\tau = \mu A_\tau^\mu Z_\tau^{1-\mu} L_\tau^{\mu-1} \tag{57}$$

as the marginal revenue product of labour (MRPL) at time τ, the following are obtained as necessary conditions for profit maximization:

$$E_t \left\{ \int_t^\infty (\eta_\tau - w) e^{-(r+\delta)(\tau-t)} d\tau \right\} = -F \text{ if } dX_t < 0 \tag{58}$$

$$-F < E_t \left\{ \int_t^\infty (\eta_\tau - w) e^{-(r+\delta)(\tau-t)} d\tau \right\} < H \text{ if } dX_t = 0 \tag{59}$$

$$E_t \left\{ \int_t^\infty (\eta_\tau - w) e^{-(r+\delta)(\tau-t)} d\tau \right\} = H \text{ if } dX_t > 0 \tag{60}$$

The wording behind the expressions above is the following: when firing in (58) the firm equates the discounted expected marginal revenue product of labour given up by dismissing a worker, to the discounted wage cost saved by doing so, subtracting the dismissal cost paid today. When hiring in (60) the firm equates the discounted expected MRPL that the newly hired worker will provide to the discounted wage cost plus the hiring cost today. In other words Bentolila and Bertola derive that the firm optimally hires a worker whenever

MRPL reaches a constant upper barrier and fires a worker whenever MRPL reaches a constant lower barrier[5].

Bentolila and Bertola's (1990) model above is strongly linked to the random growth model of individual employment that Teulings and Van der Ende (2000) developed and that will constitute the basis of our study in this paper. We will see that the hiring costs at the firm level model can be identified as specific investments costs at the individual level. There is consequently a one-to-one correspondence between the two models if a Last-in-First-Out (LIFO) separation rules applies, see Kuhn (1988) or Kuhn and Roberts (1989) for reasons of using this model in the context of unionized firms.

3 Background on wage rates and returns to job tenure

3.1 Empirical evidence on the evolution of individual wages

In this section we put forward in more detail some empirical reports on the behavior of individual earnings. The relevant results in the studies by Abowd and Card (1989) and Topel and Ward (1992) will be shortly reviewed.

Abowd and Card (1989) present an empirical analysis of individual earnings and hours data. The paper summarizes the main features of the covariance structure of earnings and hours changes and compares it with a structure implied by a simple version of the life-cycle labor supply model. The authors use three different longitudinal surveys, two samples from the Panel Study of Income Dynamics (PSID), a sample of older men from the National Longitudinal Survey of Men 49-59 (NLS) and a sample from the control group of the Seattle and Denver Income Maintenance Experiment (SIME/DIME). It is found that a relatively simple components-of-

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[5] Bentolila and Bertola (1990) show that the value function is in fact bounded as long as $r > \vartheta_z + \vartheta_a \frac{\mu}{1-\mu}$, with the parameters defined in (52)-(55) and $H + F \geqq 0$, where H and F have been defined in (58)-(60).

variance model explains the data from all three surveys and that contrary to the life-cycle model prediction that individual productivity leads to changes in earnings and hours with a larger effect on earnings, the main source of shared variation in earnings and hours represents changes at fixed hourly wage rates.

In order to control for differences in terms of labour force experience within and between the samples, the covariances between the changes in the logarithms of annual earnings and annual hours are computed using the residuals from unrestricted multivariate regressions of changes in earnings and hours on time period dummies and potential experience. The data characteristics are not significantly affected by this adjustment, the explanatory power of the experience regressions being negligible in the each sample. For each of the four samples in the three surveys analyzed, there is a remarkable similarity between the estimated covariance structures. Contemporaneous changes in earnings and hours are significantly positively correlated although the implied[6] correlation between changes in hours and changes in average hourly earnings is negative. Consecutive changes in earnings and hours, on the other hand, are strongly negatively correlated, this indicating the presence of random measurement error in the levels of both variables. In addition something of apart importance for our purpose is that changes in earnings and hours more than two periods apart are uncorrelated in all four samples. At the same time strong evidence of nonstationarity in the covariance of earnings and hours is found. These last observations suggest that changes in earnings and changes in hours may be adequately summarized by a nonstationary bivariate second order moving average process (MA(2))[7]. Namely, the changes in earnings can in this case

[6] Abowd and Card note that since the log of average hourly earnings is simply the difference in logs of annual earnings and annual hours, the covariance between changes in hours and changes in average hourly earnings is the difference between the covariance of earnings and hours and the variance of hours.

[7] Abowd and Card (1989) define as bivariate stationary MA(2) representation of the changes in hours and earnings, a representation where $cov[\Delta \log g_{it}, \Delta \log g_{it-j}]$, $cov[\Delta \log h_{it}, \Delta \log h_{it-j}]$ and $cov[\Delta \log g_{it},$

be written as :

$$\Delta \log g_t = \varepsilon_t - b_1\varepsilon_{t-1} - b_2\varepsilon_{t-2} \tag{61}$$

where ε_t is serially uncorrelated so that $Cov[\varepsilon_t, \varepsilon_s] = 0$ for all $s \neq t$. A comparison with previous literature dealing with the covariance structure of earnings such as Lillard and Weiss (1979), Hause (1980) or MaCurdy (1982), reveals that indeed negative serial correlation between consecutive changes in log earnings is a pervasive phenomenon, that a bivariate MA(2) moving average process seems to be adequate for describing the data (with an exception in Lillard and Weiss who find significant large higher-order autocovariances of earnings) and that nonstationarity appears to be the rule (although MaCurdy finds that a stationary MA(2) process successfully summarizes his covariance structure of earnings).

The authors subsequently examine three statistical models that could be generators for the structure of earnings and hours changes above. A components-of-variance model with three sources of earnings and hours variation best fits the covariance structure of the data from each survey. The three model components are:

- time-stationary serially uncorrelated measurement error;

-a shared component of earnings and hours variation that affects the contemporaneous variances and the first and second order covariances of both earnings and hours;

- a time-varying component that affects only the variances and contemporaneous covariances of earnings and hours changes.

Abowd and Card interpret the outcome using the life-cycle labor supply as comparison framework. In this respect they associate the common nonstationary part of their model to individual productivity variation. This assumption is very interesting particularly given its link with the model to be discussed in this paper, where we also assume a nonstationary behavior of the individual productivity, although we start from different considerations. According to the

$\Delta \log h_{it-j}]$ are constant for all t and are zero for $|j| > 2$.

life-cycle model, the variation in the individual productivity affects earnings more than hours; Abowd and Card's (1989) empirical findings show however that the earnings and hours covary proportionally, which casts doubt on the labor supply interpretation of earnings and hours variation and puts forward the view that most changes in earnings and hours occur at fixed hourly wage rates.

Another interesting paper inasmuch as the individual wage evolution is concerned, but also in terms of tenure distribution and separation behavior, is Topel and Ward (1992). Topel and Ward estimate the processes of job mobility and wage growth among young American workers. The analysis is carried out on the Longitudinal Employee-Employer Data (LEED); this dataset embodies a large sample of individual labor market histories taken from Social Security earnings records: 10,000 young men are observed from the very moment of their entry in the labor market up to 15 years of quarterly post-entry labor market experience. We will be reviewing Topel and Ward's findings on the wage growth within and between jobs.

For the within-job framework, Topel and Ward use as prototype model a simplified form of a common human capital earnings function:

$$w_{jt} = H(X_{jt}, T_{jt}) + \phi_j + \epsilon_{jt} \tag{62}$$

where X_{jt} is labor market experience and T_{jt} is current job tenure on job j at time t; the function $H(.)$ is quadratic in its arguments; ϕ_j denotes an unobserved fixed effect specific to a particular job j; ϵ_{jt} is a time-varying random component of measured earnings. Having as target the estimation of the determinants of wage growth we notice that we can difference (62) within jobs and eliminate fixed effects:

$$\Delta w_{jt} = \Delta H(X_{jt}, T_{jt}) + \Delta\epsilon_{jt} \tag{63}$$

Least squares will yield unbiased estimates of the parameters of $\Delta H(.)$ if $E(\Delta\epsilon|X, T) = 0$ [8].

[8] Even if this orthogonality assumption is met however, separate effecs of tenure and experience cannot be disentangled, having $\Delta X = \Delta T = 1$ within jobs.

In order to analyze mobility decisions the autocovariance structure of the residuals $\Delta\epsilon$ is essential. Topel and Ward give one example in this sense: if $\Delta\epsilon$ were found to be i.i.d., then the evolution of wages within a job would be a random walk with drift. Following the empirical analysis, the authors find that there is a strong negative autocorrelation in $\Delta\epsilon$ at lag one, followed by small, though uniformly negative correlations at higher lags. Topel and Ward also note that the data exhibits a weak nonstationarity in the sense of declining variance of $\Delta\epsilon_{jt}$. The revealed pattern suggests an ARMA model of wage innovations. Decomposing the innovation in a systematic shock to wages e and a purely transitory disturbance η we achieve

$$\epsilon_{jt} = e_{jt} + \eta_{jt} \tag{64}$$

where e is an AR(1) process with parameter $\rho > 0$:

$$e_{jt} = \rho e_{jt-1} + \nu_{jt} \tag{65}$$

Labeling $\sigma_{\nu\nu}$ and $\sigma_{\eta\eta}$ the variances of innovations to e and respectively η and defining $C_k \equiv E(\Delta\epsilon_t \Delta\epsilon_{t-k})$, the autocovariances of $\Delta\epsilon$ will be:

$$\begin{array}{c} C_0 = \frac{2\sigma_{\nu\nu}}{1+\rho} + 2\sigma_{\eta\eta} \\ C_1 = -\sigma_{\nu\nu}\frac{1-\rho}{1+\rho} - \sigma_{\eta\eta} < 0 \\ C_2 = -\sigma_{\nu\nu}\rho\frac{1-\rho}{1+\rho} < 0 \\ C_k = -\sigma_{\nu\nu}\rho^{k-1}\frac{1-\rho}{1+\rho} < 0 \end{array} \tag{66}$$

The empirical covariance structure is described by the three-parameters set $\theta = (\rho, \sigma_{\nu\nu}, \sigma_{\eta\eta})$. In order to estimate θ, denote $F(\theta) = C = (C_0, C_1, ..., C_k)'$. The estimated covariances will satisfy $\widehat{C} - F(\theta) \sim N(0, \Sigma)$ and the problem has been reduced to finding $\widehat{\theta}$, an estimate of θ, such that the quadratic $S = (\widehat{C} - F(\widehat{\theta})'\Sigma^{-1}(\widehat{C} - F(\widehat{\theta}))$ is minimized. Topel and Ward (1992) use a method of moments estimator for θ, subsequent to minimizing S and expanding $F(\widehat{\theta})$ about an initial consistent estimate θ_0 as follows:

$$\widehat{\theta} - \theta_0 = \left[F'(\theta_0)^T\Sigma^{-1}F'(\theta_0)\right]^{-1} F'(\theta_0)\Sigma^{-1}(\widehat{C} - F(\theta_0)) \tag{67}$$

Following the empirical estimation, Topel and Ward conclude that the evolution of within-jobs earnings is approximated by a random walk with drift. Namely, they obtain an estimate of $\widehat{\rho} = 0.97$, not materially different from unity. This result is similar to the result of Abowd and Card (1989), paper shortly reviewed above; the main difference is that the computed variances in the latter study are greater. Topel and Ward explain this in the light of the greater measurement error in the survey data used by Abowd and Card. An immediate consequence to this empirical finding is that past wage innovations do not predict future wage growth and therefore the current wage, experience and tenure are sufficient statistics for the distribution of future wages on a particular job. In particular heterogeneity among jobs should not be an essential feature of the data in predicting wage growth. They additionally remark that the estimated variances of e and η suggest a 95% of the within-job residual variance associated with the permanent component e, the rest being due to either measurement error or transitory shocks to measured earnings.

The authors admit that the problem with their empirical approach is the strong assumption of no correlation between the change in the unobserved component and the observables. That is, the initial assumption of orthogonality between the wage innovations and the experience and tenure, $E(\Delta\epsilon|X,T) = 0$, might be violated in case of sample selection. More precisely, if current mobility decisions are affected by the current innovations to the within-job offers then the wage outcomes are observed only for the individuals who do not change jobs. Henceforth the wage growth measured for the group of stayers will in fact be an overestimate of the potential wage growth available for any worker.

Topel and Ward also look at between-job wage growth. They study transitions to jobs that survive for at least one quarter in the LEED sample (implying that jobs lasting a quarter or less will be seen as elements of a single transition). Briefly describing their approach, we note that the wage growth estimate for the between-job transition from job $j-1$ to job j is

$$E(w_{jt} - w_{j-1,t-1} | w_{j,t+1}, w_{j-1,t-2}) = w_{j,t+1} - w_{j-1,t-2} \\ - E(w_{j,t+1} - w|.) - E(w_{j-1,t-1} - w_{j-1,t-2}|.) \tag{68}$$

In the expression above the last two terms denote the expected wage growth on the new job j and the old job $j-1$, respectively.

A few relevant empirical facts obtained using (68) are enumerated next. The typical job change during the first career stage is associated with a 12% increase in the individual's quarterly wage, compared to average quarterly wage growth of only 1.75% within jobs. Looking at the determinants of wage changes at job transitions, one of the conclusions is that between-job wage gains decline with experience and with prior job tenure. Moreover average wage gains are largest in transitions to more durable jobs; an increase of one year in completed job duration is for instance associated with 1% point increase in the initial wage of a new job. Although no causal relationship can be directly inferred (given the endogeneity of the mobility decision), these outcomes suggest that in general workers' mobility decisions are strongly affected by the job-specific wage so these gains are in fact a key element in generating the sorting to stable employment relations.

The second part of Topel and Ward (1992) is reserved to theoretical work. The authors build a model to encompass all the empirical findings mentioned above; they use a framework of mobility decisions based on wealth-maximizing on the job-search (similarities with the more recent framework in Burdett and Mortensen (1998) can easily be noticed). We shall be less concerned with the specifications of this model given that our main purpose was to revisit their empirical upshots. Nonetheless we will discuss some of the main assumptions, particularly emphasizing those linked to the theoretical framework. Wage offers from potential new employers are generated by a known offer distribution. In a homogenous workers' world, the accumulation of the general human capital will influence the location of the external wage offer distribution, which depends on the worker's accumulated labour market experience X:

$$prob(w^0 < z; X) = G(z; X), \text{ with } G_x(z; X) \leq 0 \tag{69}$$

In particular, looking at (69), external wage offers increase with experience if $G_x(z; X) < 0$ although observed wages will increase to experience due to search even in the case where productivity is independent of experience, i.e. $G_x(z; X) = 0$. What is more interesting is the way Topel and Ward accommodate for the geometric Brownian pattern of the wages, observed empirically. They assume that the probability distribution of a new internal wage offers, w^i, from the current employer, depends on the current wage, experience and tenure:

$$prob(w^i < y; w, X, T) = F(y; w, X, T) \tag{70}$$

The within-job wage growth is consequently described by the triplet (w, X, T), which is a sufficient statistic for the distribution of current and future wages on a job. A higher current wage triggers a higher distribution of future offers, $F_w(.) < 0$ and, in the case where the expected wage growth is non-increasing with experience and tenure, we have $F_x(.) \geqslant 0$ and $F_T(.) \geqslant 0$. Both the internal and the external wage offers are to be drawn from a Poisson distribution with parameter π.

The wage distributions summarized in (69) and (70) imply a value function $V(w, X, T)$, which gives the present discounted value of lifetime wealth from optimally searching on a job currently paying wage w. A job change will occur if the new job with zero tenure offers greater expected wealth than the current job, i.e. $V(w, X, T) < V(w^0, X, 0)$. A reservation offer $R(w, X, T)$ is thus defined such that any external offer exceeding this level is acceptable. Therefore, given π as probability of receiving a new offer, the hazard of leaving a job at tenure T, conditional on the fact that the worker has not left before T is:

$$\lambda(w, X, T) = \pi prob(w^0 > R(w, X, T)) = \pi[1 - G(R(w, X, T); X)] \tag{71}$$

Topel and Ward note that if $R(.)$ is differentiable then the effects of observables on mobility are:

$$\lambda_w(w, X, T) = -\pi g(R; X) R_w(w, X, T) \tag{72}$$

$$\lambda_T(w, X, T) = -\pi g(R; X) R_T(w, X, T) \tag{73}$$

$$\lambda_X(w, X, T) = -\pi g(R; X) R_X(w, X, T) - \pi G_X(R; X) \tag{74}$$

where $g(.)$ is the density of wage offers, with $g(z; X) = G_z(z, X)$.

To start with, it is clear from (72) that $\lambda_w(.) < 0$, formalizing the intuition that a higher wage increases the value of the current job and as a consequence it also increases the reservation offer; this further implies that the job is less likely to end. Second, the sign of $\lambda_T(.)$ in (73) depends however on $V_T(.)$. If one assumes for instance that the wage growth is larger at the beginning of tenure (as Topel and Ward observe empirically), i.e. $F_T(.) > 0$, then $\frac{\partial V(w,X,T)}{\partial T} \leq 0$ and $R(w, X, T) < w$ for $T > 0$. The implication is that controlling for the wage, new jobs are more valuable since they do offer higher expected wage growth and this would determine workers to accept an eventual wage cut so that they obtain these new jobs. Mobility increases thus with tenure, conditional on the current wage and experience, if expected on the job wage growth is declining. Finally, in order to discuss the sign of $\lambda_X(.)$ from (74), we need to consider both the effect of X on the reservation offer and on the distribution of alternatives. If wage offers increase with experience (typical in search models), i.e. $G_X(.) < 0$, then the effect on the distribution of alternatives will be positive. However this effect cannot be identified since more accumulated experience affects wealth on both the current as well as on the alternative jobs. What one can state however is that the current job is more likely to end as experience accumulates, since R is independent of the experience at $T = 0$: $R(w, X, 0) = w$, so (74) implies actually that mobility is increasing with experience, i.e. $\lambda_w(w, X, 0) > 0$. An important conclusion that Topel and Ward draw from this is that in a homogeneous setting where wages are controlled for, a worker with more experience is employed in a poorer match relative to his alternatives; this suggests that a direct test of the hypothesis that wage offers rise with experience is that mobility needs to rise with experience, conditional on the current tenure and wage.

The theoretical analysis brings about more insight to the pure empirical analysis Topel and Ward do in the beginning of their study. One key implication following the estimation of the theoretical framework is that while the unconditional job exit hazard may decline in both labor market experience and current job tenure as found empirically, these effects are actually reversed when conditioning on the current wage. Moreover, conditional on the current wage, a higher starting wage does influence the increase in mobility, although the current wage dominates by far in mobility decisions. As a further step it is found that the job-specific wage growth matters: jobs offering higher wage growth are significantly less likely to end, holding the current wage fixed. As the authors note, this finding would seem reasonable if jobs differ systematically in their prospects for growth, but it comes as a puzzle given that the pure empirical analysis found the within-jobs wage evolving according to a random walk.

As concluding part to this subsection we shall put together the main results in Topel and Ward (1992). One empirical outcome of the study is that the starting career phase is characterized by a pattern of rapid wage growth and high turnover. It is found in this regard that 66% of all new jobs among young workers end in the first year, with the typical young worker holding 7 full-time jobs during his first 10 years in the labor market; furthermore, wages grow extremely rapidly during the early career phase, averaging over 11% annually in the data used. In particular the wage gains when the subject changes jobs average 10% and they account for one third of total wage growth in the first 10 years on the labor market. A second empirical finding is that the evolution of wages within jobs closely approximates a random walk with drift and thus the past wage innovations do not have any say in the future wage growth. On the other hand, applying the theoretical framework developed, Topel and Ward checked that the job-change behavior of the young workers observed empirically is consistent with matching models of on-the-job search; in particular controlling for unobserved heterogeneity, the essential element leading to the eventual durability of jobs is the wage. Furthermore the wage growth is largely an outcome of the search process itself; good matches tend to survive and the decline in average mobility as experience accumulates is mainly attributable to locating such a match.

3.2 Tenure profiles in wages? A literature overview

The importance of correctly measuring the returns to seniority cannot be underestimated, knowing that quite a few major streams in the literature strongly relay on such outcomes, yielding explicit implications about the wage-tenure relationship. Thus, albeit we are interested in assessing the empirical relevance of theories of human capital, job matching, search, incentive structures or insurance motives, qualifying and quantifying the impact of job tenure on individual earnings is crucial. We are however far from having the slightest clear image of whether seniority causes increases in wage, let alone finding the magnitude of such an effect. This has been a very controversial issue in labour economic literature ever since large panel data sets became available about three decades ago, boosting the start of meaningful research in the field. A few pioneering studies among which Bartel and Borjas (1981), Borjas (1981) or Mincer and Jovanovic (1981), concluded that there is a large return to seniority on the basis of running simple OLS regressions and finding strong positive significant relationship between tenure and wage rates in cross-section or cross section-time series data A chain of papers published a few years later, e.g. Altonji and Shakotko (1987), Abraham and Farber (1987) or Williams (1991), challenged these results by contesting the adequacy of simple least squares estimates and claiming at their turn to have proved that the wage returns to seniority are relatively small. Topel (1991) re-analyses the previous literature and argues that the above-mentioned class of studies are flawed because they use inappropriate methods. He finds again high returns to tenure. Yet another influential paper by Altonji and Williams (1997) revisits the earlier literature and opens once again the debate arguing that Topel's methodology leads to biased estimates; they conclude that the tenure profiles on wages are much smaller than predicted by Topel and close to the ones predicted by Altonji and Shakotko or Abraham and Farber. Farber (1999) presents a good survey of the previous literature, raising among other things a few questions over the validity of the estimators in the studies finding modest returns to seniority. Whether wages increase with tenure, tenure increases with wages or whether the relationship between tenure and wages is too ambiguous, remains an open issue and we shall not try to debunk this. What we will do is present in detail in this section a synopsis of the debate and bring to attention in the next section an alternative view: a non-deterministic tenure profile generated by the specifications of a random growth productivity model.

Before we begin the literature visiting-tour it is compelling to note where does the uncertainty around estimating returns to seniority come from. The problem originates in job seniority being endogenous, i.e. being an outcome of optimization decisions by workers and firms. Hence, while employees with different levels of tenure might receive different wages, this does not necessarily imply a causal effect between seniority and wages. It may simply underlie the fact that both job tenure and wages are determined by some similar unobservables. This argument is also the separating front between two streams of competing theories in the area. On the one hand, there are several theories implying that wages depend on job tenure as well as on labor market experience. As mentioned in the beginning of this subsection, one of these is the human capital theory; the basis of this idea is that additional years of experience imply greater accumulated general human capital further resulting in higher wages in all jobs. Moreover, as tenure increases, job-specific skills are enhanced, leading to an increase of the wage in the current job. On the other hand, there are a series of alternative explanations for wage, tenure and experience that rely on incomplete information about the employee-employer match or about the unobserved characteristics of the worker, such as ability. Thus uncertainty over a match is reduced over time and wages adjust to reflect true productivity; in this respect good worker-firm matches survive, while a bad match causes separation. In these alternative models, wages do not rise due to accumulation of tenure or experience per se, but rather because of better allocation of workers to jobs given better information about true productivity (such an explanation is for instance one of the outcomes in Topel and Ward (1992)).

Estimation-wise, the job tenure endogeneity played the main role in the initial debate over returns to seniority: literature dating back to papers by Borjas (1981) or Mincer and Jovanovic (1981) has estimated these returns using standard earnings functions and simple OLS regressions. This approach found large magnitudes in wage raise due to additional years to tenure, in the order of 10% to 20% per 10 years of accumulated tenure, depending on the exact specification. More recent literature brought up worries about biases that may affect OLS estimates since job tenure is endogenous and cannot be assumed to be an independent source of wage variation. Inter alia, Altonji and Shakotko (1987) argue that the estimated return to job tenure in such research is biased upward because tenure is correlated with omitted individual or job factors also correlated with earnings. In essence more stable workers or workers in more stable jobs are likely to be more productive workers or respectively, on more productive jobs. Assume that the wage of individual i in job j at period t is determined as

$$\ln W_{ijt} = \beta_0 + \beta_1 X_{ijt} + \beta_2 T_{ijt} + \varepsilon_{ijt} \tag{75}$$

where X_{ijt} is a vector of characteristics of the person and the job, including for instance experience (without loss of generality we will assume here that X_{ijt} is in fact only labour market experience), T_{ijt} is tenure and, essentially, ε_{ijt} is the error term further decomposed as

$$\varepsilon_{ijt} = \lambda_i + \delta_{ij} + \eta_{ijt} \tag{76}$$

In (76) λ_i is the individual specific effect, δ_{ij} is a fixed job match effect and η_{ijt} is a transitory component. Altonji and Shakotko contend that the tenure variable T is likely to be correlated with both the individual specific and the job match effect inducing an upward biased estimate of the return to tenure β_2. The following instances are mentioned by the authors to illustrate their claim: T_{ijt} and λ_i are prone to be correlated since high productivity individuals are expected to receive high wages and to be less likely to experience a job displacement; moreover individual attributes such as health problems (alcoholism) and lack of perseverance would probably be positively correlated with separation and negatively correlated with tenure, productivity and wages. Next, T_{ijt} and δ_{ij} in (75) and (76) are expected to be also correlated. First, workers receiving higher wages compared to their alternatives will not have an incentive to quit, resulting in a positive correlation between wages and tenure in a cross-section. Second, a positive correlation between tenure and wages would be induced by the correlation between match heterogeneity in the layoff probability and in the wage equation, respectively. Finally workers quit to a better job if and only if the alternative is sufficiently high to compensate for the effect of wages on lost tenure and mobility costs, inducing a negative correlation. The net effect of these effects is expected to cause an additional upward bias in simple OLS estimates. Once the endogeneity problem revealed, Altonji and Shakotko (1987) propose a solution using an instrumental variables (IV) strategy. Using 1968- 1981 Panel Study of Income Dynamics (PSID) on earnings over time, they construct for each worker-job pair and each year the deviation of current job tenure from the mean observed job tenure of this match. The deviation is used as an instrument to identify the true effect of tenure on wages. Their results indicate that tenure has very modest effect on wage growth. Altonji and Shakotko's preferred estimates are that 10 years of tenure are responsible for a wage increase of 6.6%[9], compared to their simulated OLS estimates which indicate a 30% wage increase over 10 years or to OLS estimates in early studies, of 10-20% wage gains per 10 years. Accumulated labour market experience accounts for most of the wage growth during a career.

Abraham and Farber (1987) acknowledge as well the dilemma with the standard OLS estimates, but they expand on it in a slightly different way, proposing a different solution than Altonji and Shakotko's. In Abraham and Farber the individual and the job specific error components of equations (75)-(76) are correlated with completed job duration, tenure being only indirectly affected through the completed duration. Intuitively this argument is based on the observation that workers with long tenure must be in long jobs, while workers with short tenure can be in either short or in long jobs. Abraham and Farber also use data from the PSID, estimating separate regressions for subsamples of white-collar, respectively blue-collar workers. Abraham and Farber estimate an augmented earnings function[10] to (75)(where (76) still holds)

$$\ln W_{ijt} = \beta_0 + \beta_1 X_{ijt} + \beta_2 T_{ijt} + \beta_3 D_{ij} + \varepsilon_{ijt} \tag{77}$$

with D_{ij} representing imputed completed job duration for worker i on job j. Since completed job durations are right-censored for many jobs in the sample, a parametric model of job duration is used in the estimation of these censored observations. The parametric model is further used in computing an estimate of expected completed job duration conditional on the job lasting at least as long as the last

[9] Depending on the exact IV specification, Altonji and Shakotko obtain actually percentage gains in wages ranging from 2.7% to 11.1% for an accumulated tenure of 10 years (to be noted that the upper bound is still considerably smaller compared to simple least squares estimates); they prefer the 6.6% estimate.

[10] The precise format of the earnings function used is slightly different than the one in the original paper by Abraham and Farber (1987), being analogous to the one used in the survey provided in Farber (1999); this however does not change at all the interpretation of their methodology and results

observed seniority level. Thus for censored spells the latter estimate is used, while for non-censored ones the authors use the actual completed job duration. Abraham and Farber present an instrumental variable approach as well using the fact that in a cross-section, tenure is on average half of completed job duration. This means that the residual from a regression of tenure on completed job duration can be used to instrument tenure. Both this IV approach and the estimation of equation (77) yield results that are much smaller than OLS estimates and are close to the estimates of Altonji and Shakotko (1987). Abraham and Farber obtain a return to seniority of approximately 5% per 10 years for white-collar workers and 2.5% per 10 years for the blue-collar subsample.

A different approach than the one featured in the previous two papers has been undertaken by Topel (1991). Topel adopts an adverse position, arguing that there are in fact substantial returns to tenure even when bias originating in tenure endogeneity is being taking care of. He uses a 2-stage estimation procedure that yields a lower bound estimate of the return to tenure. The wage function is slightly modified, so that instead of using accumulated labour market experience (X_{ijt}), Topel uses the labour market experience at the start of the job (X^0_{ijt}) in the wage equation (75):

$$\ln W_{ijt} = \beta_0 + \beta_1 X^0_{ijt} + \beta_2 T_{ijt} + \varepsilon_{ijt} \tag{78}$$

The relationship between cumulated and initial experience is given by:

$$X_{ijt} = X^0_{ijt} + T_{ijt} \tag{79}$$

implying that controlling for current labour market experience, the estimated return to tenure in (78) is in fact $\beta_2 - \beta_1$. The return to initial experience, β_1, is subtracted from β_2, which reflects wage growth due to both tenure and experience accumulation. Topel conjectures that what he estimates in this way is an unbiased estimate of β_2 and a potentially upward biased estimate of β_1; the difference $\beta_2 - \beta_1$is then a lower bound estimate of the return to tenure. The data used for carrying out the empirical tests is a sample containing the first 16 waves (1968 to 1983) of the PSID. The first stage of Topel's estimation procedure is obtaining an unbiased estimate of β_2 using the average within-job wage growth of the workers who do not change jobs; he assumes that selection biases arising in this subsample of stayers can be neglected and he relies in this sense on the fact that earnings follow a random walk pattern once the growth trend is removed, so that the current change in earnings is unrelated to past changes in earnings (as we have seen Abowd and Card (1989) or Topel and Ward (1992) presented positive evidence in this sense). The second stage of Topel's estimation procedure consists in using the estimate $\widehat{\beta_2}$ from the first stage in a second regression, in order to derive the estimate of the return to experience β_1. To this aim he estimates a variation of expression (78), namely

$$\ln W_{ijt} - \widehat{\beta_2} T_{ijt} = \beta_0 + \beta_1 X^0_{ijt} + \varepsilon_{ijt} \tag{80}$$

The contention is that $\widehat{\beta_1}$ from the regression above is an upward bound on the impact of labor market experience on wages. In terms of outcomes, this 2-step estimation procedure provides Topel with returns to tenure in the order of 25% wage raise over an accumulated period of 10 years of tenure. This magnitude is in fact close to early studies that estimated the returns to seniority using simple OLS techniques.

Altonji and Williams (1997) investigate in detail relevant earlier work, focusing with predilection on issues of timing, measurement and specification. In an attempt to reconcile the results in Altonji and Shakotko (1987) with Topel's (1991) results, Altonji and Williams argue that Topel's methodology is strongly biasing upward the returns in his sample. They claim that the divergent results between Altonji and Shakotko's study and Topel's approach consist in Topel's use of the lagged wage with the current tenure as detrending procedure and in the differences between the estimators applied in the two studies. In essence, Altonji and Williams (1997) conclude that while Altonji and Shakotko's results are biased downward by unobserved job match heterogeneity, Topel's results are considerably biased upward due to individual heterogeneity in his estimator. These facts can be emphasized when the trend in the PSID is properly accounted for and when the timing of the wage and tenure measures are consistent. The authors conclude that both Altonji and Shakotko (1987) and Topel (1991) should actually give an approximate wage-tenure profile of slightly more than 10% wage increase in a 10 years period, hence close to Altonji and Shakotko's (1987) preferred estimate and far lower than Topel's (1991). Attention is also brought to issues of measurement error in tenure, the authors revisiting the assumptions made in previous research. Additionally, using survey wage rates rather than average hourly earnings, Altonji and Williams push the 10% estimate downward to only 6% wage growth in 10 years of cumulated tenure, which comes even closer to the estimate in Altonji and Shakotko. The overall conclusion of this study is once again that returns to job seniority play a modest role in determining wages.

The debate over tenure profiles continues to date. Questions over the validity of the estimators in Altonji and Shakotko (1987), Abraham and Farber (1987) and Altonji and Williams (1997), have been stressed again. Farber (1999) argues that controlling for current tenure, the completed or mean tenure possibly depend on factors that are correlated with the current wage. In addition there might be certain cases when a typical assumption in all these studies that job seniority should be a linear function of the unobserved determinant of wages, does not hold. Consider for instance a hypothetical scenario where quits occur when current wages are lower than a given exogenous threshold and the unobserved determinant of current wages affects tenure only indirectly, by affecting quits. Then job tenure would be a non-linear function of any unobserved determinant of current wages. Furthermore, recent empirical research casts once again doubt on studies finding small returns to tenure, keeping the controversy alive. A literature stream that for reasons of space has been totally overlooked in the discussion above is evaluating returns to tenure using data on workers displaced for reasons exogenous to their own or to their employers. Within this stream a few recent papers have found very substantial significant returns to tenure. Using German administrative data on displaced workers due to firm closure, Dustman and Meghir (2003) find average returns to firm tenure in the order of 20% for skilled workers and 35% for unskilled workers for a ten year period of tenure. Their methodology relies on the assumption that age affects the probability of finding a post-displacement job but not the level of that wage offer and is carried out in two-stage estimation approach in the spirit of Topel (1991). Clearly their results are close to the estimates of Topel (1991) and very much different than the ones in studies finding modest returns to seniority. On the same line, a study by Givord and Maurin (2003) on French labour force survey data, focusing on involuntarily displaced workers and using as instrument the relative number of dependent children of the workers, find average returns to tenure of 35% over 10 years of tenure. Next to these studies using data on displaced workers, a recent research on participation, job mobility and returns to seniority by Buchinsky et al (2001) explicitly specifies a framework allowing for discrete changes in starting wages in a joint model of participation and mobility. They estimate the model using as methodology Bayesian analysis with extensive Monte-Carlo methods to compute the posterior distribution of the model's parameters. The estimation is carried out on a PSID sample for separate educational groups, including high-school dropouts, high-school graduates and college graduates. The values obtained are close, on average, to Topel's outcomes; they are larger at high levels of tenure although the authors blame this on their lack of use of squared and higher orders of tenure in the estimation regression, as Topel does.

4 The Model

4.1 Predictions on seniority pattern, exit threshold and tenure profiles in wages

We have so far discussed a series of models on job tenure determination, reviewed empirical findings on wage rate evolution and introduced the controversial subject on tenure profiles in wages. This background ought to constitute a sound overture to the model of this paper. The basic setting of the model is similar to the framework developed in Teulings and van der Ende (2000). We consider a labour market model with risk neutral agents in continuous time, where a job is a match between a worker and a firm owning a vacancy. At the start of the employment relationship specific investments are made. These investments will be lost upon separation. A job produces a particular type of output with a given market value per unit of time, P_t. While both the worker and the firm are perfectly informed about the present value of P_t, its future evolution is uncertain. We shall assume that P_t follows a geometric Brownian. The labour market is "perfect" in all other respects. There are no search costs, the displaced workers being able to find new firms immediately and the other way around. There is efficient bargaining so that no surplus is left at the moment of separation[11]; quits and layoffs are therefore observationally identical, although behaviourally distinct. Moreover firms and workers are able to deal with hold-up problems and to divide future surpluses according to their share in the specific investments. The last assumption makes sure that there is no loss of generality in treating the firm as the agent paying for all specific investments and acquiring all rents.

[11] There is evidence that runs contrary to this assumption. For instance Jacobson, LaLonde and Sullivan (1993) find that particularly high-tenure workers experience substantial earnings losses when they leave their jobs, with relatively slow rates of earnings recovery after securing a new job. Teulings and van der Ende (2000) argue however for maintaining this assumption in order to simply the model and to obtain a clear idea about the parameters consistent with a first-best world.

Denoting by $V(P_t)$ -the value of a vacancy (at time t), $J(P_t)$- the value of the filled job, k and t - arbitrary shock dates, ρ - the interest rate, T - the efficient separation rate, X - the efficient hiring date and I - specific investments, the values of a vacancy, respectively a job, are given by the functional forms:

$$V(P_t) = E_t\left[e^{-\rho(X-t)}(J(P_x) - I)\right] \tag{81}$$

and

$$J(P_t) = E_t\left[e^{-\rho(T-t)}V(P_T)\right] + E_t\left[\int_t^T e^{-\rho(k-t)}(P_k - 1)ds\right] \tag{82}$$

The intuition behind (81) is that the value of a vacancy is the option of filling the vacancy in terms of making the specific investment I at some unknown future date X. Equation (82) for the value of a filled job is made up of two parts: the first term is the option to fire the worker at some unknown future date X, in which case the firm holds the value of a vacancy; the second term represents the expected worker's productivity value from which we subtract the alternative gains the worker would make on the outside market. When switching from a vacancy to a filled job we set $t = X$ in (81) and when separating we set $t = T$ in (82). We obtain the value-matching equations:

$$\begin{cases} J(P_x) = V(P_x) + I \\ V(P_T) = J(P_T) \end{cases} \tag{83}$$

By hypothesis, $\log P_t$ is a Brownian with drift μ and variance σ^2, thus the law of motion between arbitrary dates k and t is

$$(t-k)^{-1}[\log P_t - \log P_k] \sim N[(\mu, \sigma^2) \tag{84}$$

The firm will hire a worker at the moment X when P_t hits an upper bound P_X and the worker will separate at moment T when P_t hits a lower bound P_T. The job match ends when the worker and the firm do not consider its continuation beneficial. Further define

$$\begin{array}{c} \Delta_t \equiv \frac{P_t - P_T}{\sigma} \\ \Delta \equiv \frac{P_X - P_T}{\sigma} \\ \pi \equiv \frac{\mu}{\sigma} \end{array} \tag{85}$$

where Δ_t and Δ are the normalized distances between the actual productivity, respectively the hiring moment, and the separation threshold; π is the normalized drift. The job tenures are then fully determined by two parameters, Δ and π.

Since Δ is not a structural parameter, it will depend on the optimal hiring and exit thresholds $p_X = \log P_X$ and $p_T = \log P_T$. For determining these parameters we shall use Dixit's (1989) option theory. All premises are fulfilled in order for Dixit's theory to be applied, in that the investments made are lost upon separation and hence there is an immediate analogy with financial investment decisions. One of Dixit's (1989) own examples states that "a firm that fires a worker cannot rely on hiring the same person, and must expect to train a new one should it decide to expand again", this meaning that specific training costs are incurred each time and that they are lost upon job dislocation. He associates an opportunity of making a real investment (here, the firm specific investments) with a call option on a a stock that consists of the capital in place (here, the match productivity value). Further, making this investment is similar to actually exercising this option, with its cost being the strike price of this option. Standard methods imported from financial mathematics would give the price of the option (here, the value to the firm to starting the employment relationship) and the rule that tells how to exercise the option optimally (here, the efficient bargaining assumption). Using Dixit's insight, Teulings and van der Ende (2000) identify the optimal hiring and separation timing. The application of Ito's lemma in the context of equations (81) and (82), gives value functions for a vacancy, respectively a job, described by two Bellman equations:

$$\rho V(P_t) = (\sigma\pi + \frac{\sigma^2}{2})V'(P_t)P_t + \frac{\sigma^2}{2}V''(P_t)P_t^2 \tag{86}$$

and

$$\rho J(P_t) = (\sigma\pi + \frac{\sigma^2}{2})J'(P_t)P_t + \frac{\sigma^2}{2}J''(P_t)P_t^2 + P_t - 1 \tag{87}$$

Both equations (86) and (87) have as format a first term that takes care of the drift in the productivity and a second term reflecting the second order effect of the shocks to productivity. Equation (87) has an extra term that accounts for the current output of a filled vacancy, net of the outside worker's option. Using a derivation in Dixit and Pindyck (1993) for the Bellman equations above, the hiring and the exit momenta are described by the following relations (following Teulings and van der Ende(2000))

$$\begin{aligned} I = {} & \frac{1}{\sigma\rho}\frac{\sigma\alpha_1(1-D^{\sigma-\alpha_1})(D^{\alpha_1}-D^{\alpha_1-\alpha_2})}{\alpha_1(1-D^{\sigma-\alpha_1})D^{\alpha_1-\alpha_2}+\alpha_2(D^{\sigma-\alpha_2}-1)+\frac{2\rho}{\sigma}(D^{\alpha_1-\alpha_2}-1)} \\ & +\frac{1}{\sigma\rho}\frac{\sigma\alpha_2(D^{\sigma-\alpha_2}-1)(D^{\alpha_1-1})+2\rho(D^{\sigma}-1)(D^{\alpha_1-\alpha_2}-1)}{\alpha_1(1-D^{\sigma-\alpha_1})D^{\alpha_1-\alpha_2}+\alpha_2(D^{\sigma-\alpha_2}-1)+\frac{2\rho}{\sigma}(D^{\alpha_1-\alpha_2}-1)} \end{aligned} \tag{88}$$

$$P_X = \frac{1}{\sigma}\frac{2(\rho-\pi\sigma-\frac{\sigma^2}{2})(D^{\alpha_1-\alpha_2}-1)D^{\frac{1}{\sigma}}}{\alpha_1(1-D^{\sigma-\alpha_1})D^{\alpha_1-\alpha_2}+\alpha_2(D^{\sigma-\alpha_2}-1)+\frac{2\rho}{\sigma}(D^{\alpha_1-\alpha_2}-1)} \tag{89}$$

$$P_T = \frac{1}{\sigma}\frac{2(\rho-\pi\sigma-\frac{\sigma^2}{2})(D^{\alpha_1-\alpha_2}-1)}{\alpha_1(1-D^{\sigma-\alpha_1})D^{\alpha_1-\alpha_2}+\alpha_2(D^{\sigma-\alpha_2}-1)+\frac{2\rho}{\sigma}(D^{\alpha_1-\alpha_2}-1)} \tag{90}$$

where by (85),

$$D \equiv e^{\Delta} = \left(\frac{P_X}{P_T}\right)^{\sigma} \tag{91}$$

and

$$\begin{cases} \alpha_1 \equiv -\pi + \sqrt{\pi^2 + 2\rho} \\ \alpha_1 \equiv -\pi + \sqrt{\pi^2 + 2\rho} \end{cases} \tag{92}$$

We find from this setup that p_X is positive, while p_T is negative. This means that immediately after the hiring decision the productivity in the job exceeds the outside option by at least the interest payments on the specific investment, ρ. The intuition behind this fact is simple: if this were not the case, the firm could of course increase its profits by postponing the investment. At the separation threshold, the productivity P_T is below unity (so the log productivity p_T is negative). Intuitively, this should be the case because otherwise it would be better for the firm to keep the worker rather than displace him, so that the cost of specific investment is not incurred again.

Returning to the analysis of the tenure distribution, since we identified the optimal entry and exit thresholds in (89) and (90), we can further use the Δ parameter from (85). The distribution of $\Delta_t - \Delta$ conditional on X is $N[(t-X)\pi, (t-X)\sigma^2]$. When Δ_t is negative it means that separation time has occurred at some time $X < T < t$. We are faced however with a potential dilemma in that we do not know which of the realizations of $\Delta_t > 0$ are actually associated with an ongoing employment relation. It might be the case that for some time s in the interval (X, t) we had $\Delta_s < 0$ but in fact Δ_t travelled back to a positive value since then; or this cannot be corresponding to a job-worker match since separation decisions are irreversible. What we are thus interested in is the probability that no separation occurs before time t, which is actually the conditional density of $\Delta_s > 0$ for all $X \leq s < t$. One can apply a simple methodology, very often used for instance in pricing barrier options in mathematical finance, that is appropriate in this regard, the stochastic reflection principle[12]. Having the underlying process described by a Brownian motion, the reflection principle implies for our purpose that there is a one-to-one correspondence between trajectories from Δ to Δ_t having crossed

[12] A most standard definition of the reflection principle is the following. Consider $\{B_t, t \geq 0\}$ a standard Brownian and let T be a stopping time. Then $\{\widetilde{B_t}, t \geqslant 0\}$, where

$$\widetilde{B_t} = \begin{cases} B_t, \text{ for } t \leq T \\ 2B_t - B_t, \text{ for } t > T \end{cases}$$

is also a standard Brownian. What is more interesting for us is that when $T = T_a$ then the correspondence $B_t \longmapsto \widetilde{B_t}$ amounts to the reflection of the path after the first hitting time on a in the line $x = a$.

Δ_s at least once, and trajectories from $-\Delta$ to Δ_t. This latter group should be subtracted when calculating the desired density:

$$\Pr[\Delta_t, T > t|\ t, X = 0] = \frac{1}{\sqrt{t}}\left(\phi(\frac{\Delta_t - \Delta - \pi t}{\sqrt{t}}) - \Theta\phi(\frac{\Delta_t + \Delta - \pi t}{\sqrt{t}})\right) \tag{93}$$

where $\phi(.)$ is the density and $\Theta = e^{-2\Delta\pi}$ accounts for the effect of the drift π. Teulings and van der Ende obtain the distribution of completed job tenures by integrating out Δ_t in (93):

$$\Pr[T > t|t, X = 0] \equiv 1 - F(t) = \Phi_t^+ - \Theta\Phi_t^- \tag{94}$$

where $\Phi_t^i = \Phi(x_t^i)$ and $x_t^+ = \frac{\Delta+\pi t}{\sqrt{t}}$, respectively $x_t^- = \frac{-\Delta+\pi t}{\sqrt{t}}$. More interesting is the implication on the exit rate predicted by this model. If we denote by $\lambda(t) \equiv \frac{f(t)}{1-F(t)}$ the hazard rate, the following characteristics are revealed:

$\lambda(0) = 0$ and $\lambda'(t)|_{t>0} > 0$; $\lambda(t)$ reaches its peak at t_0, with $0 < t_0 < \frac{2\Delta^2}{3}$;

$\lim_{t\to\infty}\lambda(t) = 0$ for $\pi > 0$ and $\lim_{t\to\infty}\lambda(t) = \frac{\pi^2}{2}$ for $\pi > 0$.

The shape of the hazard rate underlies the empirically observed job-exit rates, see the discussion in section 2. on various models for job tenure determination. Teulings and van der Ende (2000) compare the predicted results of the random growth model with the predictions of the random learning model. Their conclusion is that a random growth model without drift is a special case of a simplified version of the learning model but the random growth with a negative drift yields outcomes that are more consistent with empirical data than the learning model. The essential difference between the random growth and the learning framework rests in the variance of Δ which implies that while at $t = 0$ the hazard rates are identical for both models, with the passing of time the accumulation of shocks and the hazard rate in the learning model start lagging behind the one in the random growth model.

Following our discussion on the controversy over the returns to tenure in section 3, it would be of course interesting to see what the random growth model has to say in this respect. To start with, the basic model is extended with a sharing rule for the distribution of surpluses of the specific investment, rather than retaining the assumption that the firm makes all investments and captures all surpluses. This rule is given by

$$w_t = \log[1 + \beta(P_t - 1)] \approx \beta p_t = r_t + \beta\sigma\Delta_t \tag{95}$$

where the approximation has been obtained using a first order expansion that holds for small values of p_t . In (95) β represents the worker's share in paying for the specific investments ($\beta \in (0, 1)$). Using equations (93), the expected value of Δ_t for ongoing employment relations can be computed

$$E[\Delta_t | t, X < t < T, X = 0] = \pi t + \Delta \frac{\Phi_t^+ + \Theta\Phi_t^-}{\Phi_t^+ - \Theta\Phi_t^-} \tag{96}$$

The slope of the tenure profile in a cross-section regression on log wages is then equal to the derivative of this expectation with respect to t multiplied by the variance and the worker's investment share. The final expression for the slope is given by:

$$\begin{aligned} &\sigma\beta \frac{dE[\Delta_t | t, X < t < T, X = 0]}{dt} \\ &= \sigma\beta \left\{ \pi + \frac{\Delta\phi_t^+}{t\sqrt{t}[\Phi_t^+ - \Theta\Phi_t^-]} \left[\pi t + \Delta \frac{\Phi_t^+ + \Theta\Phi_t^-}{\Phi_t^+ - \Theta\Phi_t^-} \right] \right\} \end{aligned} \tag{97}$$

The result obtained indicates that for the relevant case of a negative drift $\pi < 0$ the tenure profile depends on the evolution of the match: in the short run the drift effect dominates since there is not so much selection going on, resulting in no tenure profile on wages; in the intermediate run the selection effect dominates the drift effect leading to a tenure profile on wages even if there is no inherent job specific productivity increase; in the very long run both the selection and the drift effect cancel, so we still have a tenure profile. These findings raise a question around the whole setup of previous studies on tenure profiles in wages, which try to estimate a tenure profile independent of the future perspectives on the job. If this model proves to be correct, we cannot in fact talk about a deterministic tenure profile, this depending on the evolution of a job-worker match.

4.2 Link with the firm-level random growth model and extension by including log firm size

As discussed in the introduction and in the section on models of job tenure analysis, one of the advantages of the setting in this individual random productivity model is that it can easily be related to Bertola and Bentolila's (1990) model of random growth at the firm level where there is uncertainty about future labor demand. As discussed in section 2.3, in the setup modelling the firm level employment the conclusions were that the firm starts hiring workers when the log price reaches an upper bound and starts firing when this log price reaches a certain lower bound. In fact the model of Bentolila and Bertola (1990) yields the same pattern of job seniority for individual workers as the model of individual random growth productivity, provided that the latter one is supplemented with a rule accounting for the order in which the workers are laid off. Kuhn (1988) and Kuhn and Roberts (1989) provide this rule: a last-in-first-out rule (LIFO) according to which the firm would fire first the workers with the lowest tenure on the job. This rule would also solve hold-up problems appearing because the senior workers would fear investing sufficiently since their future surpluses are endangered by claims of the newly hired employees. Protecting these incumbents by a LIFO layoff rule would prevent the firm from replacing expensive senior workers with newcomers hired cheaply on the basis of promises that they would get parts of the returns to the specific investments. Once the LIFO rule is embedded in the model at the individual level, the optimal hiring and firing thresholds for the two models coincide (Teulings and van der Ende(2000) provide a detailed analysis of this similarity).

Given the empirical results for the wage rate evolution in Abowd and Card (1989) and Topel and Ward (1992) reviewed above, as well as the contention that log firm size follows a random walk, empirically established for large firms by Jovanovic (1982) and implied by the model of Bentolila and Bertola (1990) also discussed above, we depart from the initial framework in Teulings and van der Ende (2000) generalizing it so that it takes into account the wage evolution and the firm size evolution[13]. We use the following definitions:

w_{ijt}, r_{ijt} are the log wage, respectively log reservation wage, of worker i in firm j at time t;

b_{ijt} is an indicator function such that worker i is in firm j till the first time t at which $b_{ijt} < 0$;

f_{jt} is the log firm size of firm j at time t;

Δ_{ijt}, Ω_{ijt} are the effects of specific investment on the separation threshold, respectively wages;

u_{ij}, ν_{ij} are random time invariant worker and respectively firm effects, having $[u_{ij}, \nu_{ij}]' \sim N(0, \Psi_{ij})$;

s_{ij}, q_{ij} are times t at which worker i starts working at or respectively quits from, firm j;

T_{ij} is the completed tenure of individual i at firm j so that $T_{ij} = q_{ij} - s_{ij}$

We have then as starting values for the log wage and respectively for the indicator function, the following:

$$w_{ij,s_{ij}} = r_{ij,s_{ij}} + \Omega_{ij,s_{ij}} + \nu_{ij} \tag{98}$$

$$b_{ij,s_{ij}} = \Delta_{ij,s_{ij}} + u_{ij} \tag{99}$$

The law of motion in this case will be of course more complex than the one in (84), since the variable of interest in this case follows a multivariate random walk (being a composite of random walks). Define

$$x_{it} \equiv (b_{ijt}, r_{ijt}, w_{ijt}, f_{jt})' \tag{100}$$

[13] This extension is based on suggestions of Coen Teulings

Then the law of motion will be:

$$h^{-1}[x_{i,t+h} - x_{it}] \sim N(\mu_{ij,s_{ij}}, \Sigma_{ij,s_{ij}}) \tag{101}$$

where the variance of b_{ijt} is normalized to unity, $\sigma_{bb} = 1$. The variance and drift in (101) are allowed to vary with the experience at job start but not during a job.

We are of course interested in estimating the matrix of variances, $\Sigma_{ijs_{ij}}$. The application of the methodology in Teulings and van der Ende (2000) yields a few implications about the variance-covariance matrix $\Sigma_{ijs_{ij}}$. We shall schematically outline a few of them hereinafter. The LIFO layoff rule attached above in order to make the link between this model and the random proportional growth model of Bentolila and Bertola (1990) would imply that the covariance between the duration in the job and the firm size is positive

$$\sigma_{bf} > 0 \tag{102}$$

This basically suggests that if we divide seniority in two categories, high seniority, i.e. corresponding to senior workers and low seniority, i.e. associated with "newcomers", the larger firms would fire more newcomers than senior workers or in other words that one can aspire to a longer tenure in a larger firm. Furthermore, the efficient bargaining on surpluses implies

$$\sigma_{bw} - \sigma_{br} > 0 \tag{103}$$

which says that the wages increase faster than reservation wages with seniority; in this sense we could say that the wage on tenure distribution will first order stochastically dominate the reservation wage on tenure distribution. Adding together the assumptions on the LIFO layoff rule and on the efficient wage bargaining (thus considering both results in (102), respectively (103)) gives yet a third prediction

$$\sigma_{fw} - \sigma_{fr} > 0 \tag{104}$$

on basis of which the distribution of wages controlling for firm size is stochastically dominating the distribution of alternative wages on firm size. Both (103) and (104) rely on an efficient bargaining rule between workers and firms. Finally a general implication founded on the existence of competition in this labour market is that

$$\sigma_{rw} > 0$$

which does not require further explanations. In the extended model we could moreover test for both the implication that log wages follow a random walk as well as Gibrat's law for firm size evolution.

Certainly one of the problems with this generalized model is that in practice it would be very cumbersome to estimate it. As a first stage in the estimation process, several simplifications would thus enhance feasibility. One such simplification would be holding fixed the variance and the drift of the random walks involved so that these are the same for all individuals and all firms and they do not vary with experience at the start of the job either; following the notation introduced above, $\mu_{ij,s_{ij}} = \mu$ and $\Sigma_{ij,s_{ij}} = \Sigma$, respectively. We can also assume there are no random time invariant effects, $\Psi_{ij} = 0$. A further simplification would be assuming only constant effects of specific investments (although this would certainly reduce much of the model's plausibility): $\Delta_{ijt} = \Delta$, $\Omega_{ijt} = \Omega$. The restrictions can be relaxed once the parameters of the basic model have been identified.

5 Summary and issues for further research

We have argued in this paper for the relevance of the random growth productivity model for job tenure distribution, optimal job exit rates and wage returns to seniority. In this respect an extensive review of relevant literature on models of job separation rates and seniority distribution such as search, random learning and random growth models, has been presented. The random models are better fit than the conventional job search framework in underlying the hump shaped pattern of the job exit rates observed in practice; the random growth model performs better than the random learning setup, the learning model predicting separation rates to converge to 0, whereas the random growth model generates the fat tail noticed empirically. Furthermore it was demonstrated that two of the implications of the random growth model are perfectly consistent with the empirical observations that log individual wages and respectively log firm size follow a random walk. The prediction of this model in terms of tenure profiles in wages brings new waves in the debate within the literature on the returns to seniority: if the model is sound, tenure profiles depend on the evolution of a match and it is inappropriate to talk about 'the tenure in wages', the tenure profile being necessarily non-deterministic. In terms of methodology, the random growth model presented in this paper makes use of the reflection principle to recover the tenure distribution and employs the option theory developed in Dixit (1989) in order to derive the optimal separation threshold. We have also discussed a generalized version to the model in which log firm size is inserted additionally in the model; while the methodology remains in principle the same, estimating such a model even in the simpler form would involve more complexity.

In what follows we bring to attention some further avenues for research that should be straightforward to implement. First, continuing on the discussion on the LIFO rule that was subject of a previous section, we can consider empirically testing its implications. Lindbeck and Snower (1988) put forward the labor market insider-outsider hypothesis. This stipulates that the incumbent workforce would resist to hiring additional workers, since these workers could be a threat to their own position in the firm. However this is an extreme solution to the hold-up problem and certainly leaves a scope for a Pareto improving bargain between the firm and its incumbent workers. Kuhn's (1988) or Kuhn and Robert's (1989) rationalization of the LIFO layoff rule offers the firm and its workers an instrument to realize this Pareto improvement without endangering the interests of the incumbent workers. In this respect firms may fire workers only in a particular order. Hence, incumbent workers not at the brink of being fired can ask above marginal productivity wages. Basically the model discussed in this paper would allow the extension of Kuhn's theory to a dynamic setting and its empirical testing. The rationale of such an undertaking cannot be belittled: the argument on insider power and LIFO layoff rules has important policy implications. Insider power is often associated with hysteresis and thus it is prone to have very serious microeconomic and macroeconomic consequences. A specific instance of this would be for instance the often surprising market power division between workers' unions and firms that has been long a major theme in policy discussion in most Western economies. An extension on the mentioned direction would give us better understanding of the role of LIFO rules and would allow us to evaluate whether they are really as harmful as often assumed.

Next, implications of this project are essential for issues related to insurance of workers' lifetime labour income. The random growth assumption of this model, supported as we have seen by several empirical studies, suggests that this income is subject to substantial individual specific risk. Where firms are less risk averse than workers, the optimal allocation of risk would assign most uncertainty to the firm, implying that workers would just receive their reservation wage and firms would get all the surplus. Empirical evidence suggests important differences in risk sharing between continental Europe and the United States, with workers taking much more risk in the United States. As an implication, tenure profiles tend to be much more important in the United States than in Europe. This issue has great relevance for the design of social insurance systems: where firms take care of most insurance, there is little reason for the government to provide extensive social insurance. Hence, an extension could explore the optimal division of the burden of insurance between private firms and the government. More concretely, the option theory embedded in the project would help in carrying out an efficiency-wise cross-country comparative evaluation of "standard" social insurance institutions such as unemployment and disability insurance. .

A final issue for further research is the link between the individual worker career and the evolution of the firm. This follows directly from the extension discussed in the previous section, where firm size has been included in the model. When random shocks in productivity are interpreted as shocks to the demand for the firm's product, this generalized model offers an excellent starting point for the analysis the relation of worker's career and the evolution of the firm. We have at hand all the ingredients in this sense: Teulings and Van der Ende (2000) have shown that their model for an individual worker's job tenure is embedded in Bentolila and Bertola's (1990) model of employment at the level of the firm if you extend that model with the LIFO firing rule. Kuhn (1988) and Kuhn and Roberts (1989) provided an economic argument for why that rule is to be applied. Firms that have to downsize can reduce their hiring, but often firing some workers will be inevitable. Hence, the distribution of the individual worker's job duration is intimately related to the evolution of the size of the firm's workforce. Job tenure is not to be taken as the only variable that relates a worker's odds to that of her firm. There is substantial evidence that wages of individual workers are not fully determined by their reservation wages, as suggested by simple neoclassical models, but that they are also related to the firm's profitability. Although these observations are straightforward, there is no systematic research in the precise nature of these interrelations and there is plenty of room for further investigations.

A Bibliography

Abowd, J.M. and D. Card (1989), "On the Covariance Structure of Earnings and Hours Changes", *Econometrica*, 57, Issue 2, 411-45

Abraham, K.G. and H.S. Farber (1987), "Job Duration, Seniority, and Earnings", *The American Economic Review*, 7, No.3, 278-97

Altonji, J.G. and R.A. Shakotko (1987), "Do Wages Rise with Job Seniority?", *Review of Economic Studies*, 54, No. 3, 437-59

Altonji, J.G. and N. Williams (1997), "Do Wages Rise with Job Seniority? A Reassessment", NBER Working Paper No. 6010

Bartel, A.P. and G. Borjas (1981), "Wage Growth and Job Turnover: An Empirical Analysis", in S. Rosen (ed) *Studies in Labor Markets* (NY: NBER)

Bentolila, S. and G. Bertola (1990), "Firing Costs and Labour Demand: How Bad is Eurosclerosis?", *Review of Economic Studies*, 57, No. 3, 381-402

Borjas, G. (1981), "Job Mobility and Earnings Over the Life Cycle", *Industrial and Labour Relations Review*, 34, 365-76

Buchinsky, M., D. Fougere, F. Kramarz and R. Tchernis (2001), mimeo CREST-INSEE

Burdett, K. (1978), "A Theory of Employee Search and Quits", *American Economic Review*, 68, No. 1, 212-20

Burdett, K. and D. Mortensen (1998), "Wage Differentials, Employer Size, and Unemployment", *International Economic Review*, 39, Issue 2, 257-73

Dixit, A. K. (1989), "Entry and Exit Decisions Under Uncertainty", *Journal of Political Economy*, 97, No. 3, 620-38

Dixit, A. K. (1989) and R.S. Pindyck (1993), "Investment under Uncertainty", Princeton University Press, Princeton, USA

Dustman, C. and C. Meghir (2003), "Wages, Experience and Seniority", IFS working paper W01/23

Farber, H.S. (1999), "Mobility and Strategy: The Dynamics of Job Change in Labor Markets", Handbook on Labor Economics, eds. O. Ashenfelter and D. Card, Vol. 3-B, Amsterdam, North-Holland

Feldstein, M. (1976), "Temporary Layoffs in the Theory of Unemployment", *Journal of Political Economy*, 84, No. 5, 937-58

Givord, P. and E. Maurin (2003), "Job Tenure, Wages and Technology: a Reassessment Using Matched Worker-Firm Panel Data", mimeo CREST-INSEE

Hause, J.C. (1980), "The Fine Structure of Earnings and On-The-Job Training Hypothesis", *Econometrica*, 48, No. 4, 1013-29

Jacobson, L.S., R.J. LaLonde and D.G. Sullivan (1993), "Earnings losses of displaced workers", *American Economic Review*, 83, No.4, 685-709

Jovanovic, B. (1979a), "Job Matching and the Theory of Turnover", *Journal of Political Economy*, 87, No. 5, Part 1, 972-90

Jovanovic, B. (1979b), "Firm-Specific Capital and Turnover", *Journal of Political Economy*, 87, No. 6, 1246-60

Jovanovic, B. (1982), "Selection and the Evolution of Industry", *Econometrica*, 50, No. 3, 649-70

Kuhn, P. (1988), "A Nonuniform Pricing Model of Union Wages and Employment", *Journal of Political Economy*, 96, No. 3, 473-508

Kuhn, P. and J. Robert (1989), "Seniority and Distribution in a Two-Worker Trade Union", *Quarterly Journal of Economics*, 104, No. 3, 485-505

Lancaster, T., G. Imbens and P. Dolton (1987), "Job Separations and Job Matching", in R.D.H. Heijmans and H. Neudecker (Eds.), *The Practice of Econometrics*, 31-43, Martinus Nijhoff Publishers

Lillard, L.A. and Y. Weiss (1979), "Components of Variation in Panel Earnings Data: American Scientists 1960-1970", *Econometrica*, 47, No. 2, 437-54

Lindbeck, K. and D.J. Snower (1988), “The Insider-Outsider Theory of Employment and Unemployment”, The MIT Press, Cambridge, Massachusetts

Lippman, S.A. and J.J. McCall (1976), "The Economics of Job Search: A Survey", *Economic Inquiry*, 14, 155-89, 347-68

MaCurdy, T. E. (1982), "The Use of Time-Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis", *Journal of Econometrics*, 18, 83-114

McLaughlin, K. (1991), "A Theory of Quits and Layoffs with Efficient Turnover", *Journal of Political Economy*, 99, No. 1, 1-29

Miller, R.A. (1984), "Job Matching and Occupational Choice", *Journal of Political Economy*, 92, No. 6, 1086-1120

Mincer, J. and B. Jovanovic (1981), "Labor Mobility and Wages", in *Studies in Labor Markets*, ed. S. Rosen (NY: NBER)

Mortensen, D. (1978), "Specific Capital and Labour Turnover", *Bell Journal of Economics*, 9, No. 2, 572-86

Teulings, C.N. and M. van der Ende (2000), "A Structural Model of Tenure and Specific Investments", Tinbergen Institute Discussion Paper 2000-009/3.

Topel, R.H. (1991), "Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority", *Journal of Political Economy*, 99, No. 1, 145-76

Topel, R.H. and M.P. Ward (1992), "Job Mobility and the Careers of Young Men", *Quarterly Journal of Economics*, 107, No. 2, 439-79

Van der Ende, M.A. (1997), "On the Values of Jobs and of Specific Training", Ph.D. thesis, University of Amsterdam, The Netherlands

Williams, N. (1991), "Reexamining the Wage, Tenure and Experience Relationship", *Review of Economics and Statistics*, 73, No. 3, 512-17