Rent sharing and worker-firm specific investments

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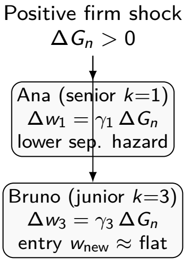
Motivation

- Within-firm rent sharing is ranked: seniors extract more.
- Entry wages move little when surplus rises; *incumbents* capture rents.
- Need a microfoundation where bargaining order matters.
- **Corrected SZ:** earlier sessions have stronger disagreement leverage.

Punchline. Common shocks imply $\Delta w_1 > \Delta w_2 > \cdots > \Delta w_n$ and $\Pr[\text{sep}_1] < \cdots < \Pr[\text{sep}_n]$.



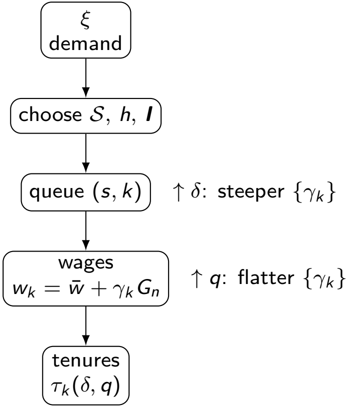
$\gamma_1 > \gamma_3 \Rightarrow$ senior paid more, fired less.

Notation: γ on slides $= \theta$ in paper.

Roadmap (theory first)

1. **Mechanism: corrected Stole–Zwiebel with fixed queue**
   Rank-specific outside options; affine wages
   $w_k = \bar{w}(\xi) + \gamma_k \, G_n(\xi)$ with $\gamma_1 > \cdots > \gamma_n$.
2. **Dynamic embedding: hire, invest, separate, pay**
   (δ, q) tilt $\{\gamma_k\}$ via continuation values.
3. **Core results: ordered shares, CS in (δ, q), weak LIFO**
   Weak LIFO without ad hoc firing costs.
4. **Sufficient statistics → estimable objects**
   Map: $(\partial w_k/\partial\xi,\ \partial s_k/\partial\xi,$ queue stats$) \leftrightarrow$
   $(\delta, q,$ bargaining primitives$)$.
5. **Empirics (preview)**: rank shocks, IV, local projections
   Design: quasi-random rank shifts trace $\{\gamma_k\}$.
6. **Robustness & alternatives**: Rolodex, Nash, unions
   Test: competing mechanisms flip key signs; ours survive.

Notation: γ on slides $=$ θ in paper.



Stylized facts (very brief)

- **Ranked rent sharing.** After positive firm shocks, $\Delta w_1 > \Delta w_2 > \cdots > \Delta w_n$; entrants' $w_{\text{new}} \approx$ flat.
- **Persistence & reshuffles.** Seniority premia persist within cohort; exogenous rank reshuffles (entries/exits/promotions) compress premia.
- **Separation hazards.** Ordered elasticities to good news: $\eta_{\Delta G}[\text{sep}_1] < \cdots < \eta_{\Delta G}[\text{sep}_n]$.
- **Against equal shares.** Equal-share benchmarks $\Rightarrow \Delta w_k$ flat in k; rejected by rank gradients and entry flatness.

Signature	Data	Equal-share
Ranked Δw_k	✓	×
Entry w_{new} flat	✓	×
Hazard gradient	✓	×

Design cues: within-firm event studies; cohort-by-rank contrasts.

Contributions

1. **Mechanism (corrected SZ).** $w_k = \bar{w}(\xi) + \gamma_k(\delta, q; \mathbf{s})\, G_n(\xi)$ with rank-specific outside options $\Rightarrow\ \gamma_1 > \cdots > \gamma_n$.
2. **Dynamic block (hire, invest, separate, pay).** Tractable Markov setting; weak LIFO emerges *without* ad hoc firing costs; clean CS in (δ, q).
3. **Identification & estimation.** Sufficient-statistics map from within-firm responses $\{\partial w_k/\partial\xi,\ \partial s_k/\partial\xi\}$ and queue moments to $(\delta, q,$ bargaining primitives$)$; estimable via rank-shock event studies + IV.
4. **Industry primitives from spells.** First-passage-time (FPT) moments identify knowledge specificity and drift; explain cross-industry seniority slopes.

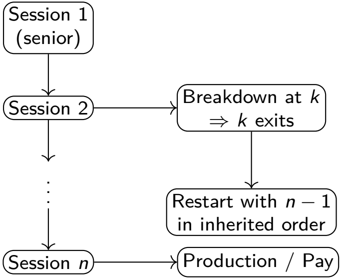
What is *new* and testable

- Closed-form, rank-ordered shares with dynamics.
- Inequalities: $\Delta w_1 > \cdots > \Delta w_n$; entry wage $\approx$ flat; ordered separation hazards.
- Sharp refutation of equal-share benchmarks via rank gradients.
- Estimation scalable to admin data; built-in falsification tests.

Deliverables: explicit γ_k, simple estimators, out-of-sample checks.

Corrected SZ: setup

- **Queue and rank.** Workers are ordered $k = 1, \ldots, n$; $k = 1$ is most senior.
- **Bargaining in session k.** Alternating offers; common discount $\delta \in (0, 1)$; per-offer breakdown $q \in (0, 1)$.
- **Breakdown rule (correction).** If session k breaks down, worker k exits; the game *restarts* with $n-1$ in inherited order.
- **Timing.** Production and pay occur *only after* all remaining sessions conclude.
- **Outside option for k.** Value if the firm continues without k; inheritance of order $\Rightarrow$ earlier ranks have stronger disagreement payoffs.
- **Surplus pieces.** Total surplus $G = G_s + G_n$, yielding $w_k = G_s + \gamma_k \, G_n$ with $\gamma_1 > \cdots > \gamma_n$ in equilibrium.



Key implication

Earlier ranks have higher disagreement leverage $\Rightarrow$ larger shares γ_k.

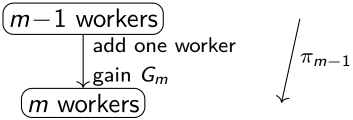
Gains from trade in the m-worker subgame

$$\underbrace{G_m(\xi)}_{\text{new trade surplus}} = \underbrace{F(m,\xi;\boldsymbol{s},\boldsymbol{\kappa})}_{\text{production with } m} - \underbrace{\pi_{m-1}(\xi)}_{\text{continuation (−1 worker)}} - \underbrace{m\,w(\xi)}_{\text{status-quo wages}}$$

- Definition. G_m is the incremental surplus from staying at m instead of falling to $m-1$ before bargaining.
- Key load-bearing object. Replacement costs, separations, and specific knowledge enter *only* via $\pi_{m-1}(\xi)$.
- *Consequence.* Comparative statics and identification reduce to movements in π_{m-1} plus the order that pins the split.

Why this matters

- Packs dynamic complexity into a single object π_{m-1}.
- Leaves the order-driven ranking of shares intact.
- Links to wages: $w_k(\xi) = w(\xi) + \gamma_k(\delta, q; \boldsymbol{s})\, G_n(\xi)$.



Corrected SZ recursion (one line)

$$\pi_k(\xi) = \pi_{k-1}(\xi) \; + \; \underbrace{(1-\gamma_k)}_{\text{firm share}} \, G_k(\xi).$$

$$w_k(\xi) = \bar{w}(\xi) \; + \quad \underbrace{\gamma_k(\delta, q; \boldsymbol{s})}_{\in(0,1),\;\; \gamma_1 > \cdots > \gamma_n} \quad G_n(\xi).$$

- Single driver. All wage blocks load the same $G_n(\xi)$; dynamics/replacement enter only via π_{m-1} inside G_k.
- Order pins shares. $(\delta, q; \boldsymbol{s})$ determine $\{\gamma_k\}$ ($\uparrow\delta$ *steepens*, $\uparrow q$ *flattens*).
- *No-delay equilibrium:* larger early-k outside options $\Rightarrow$ acceptance at γ_k and strict ordering.

Equilibrium representation

$$w_k(\xi, \boldsymbol{s}, \boldsymbol{\kappa}) = \underbrace{\bar{w}(\xi)}_{\text{common level}} + \underbrace{\gamma_k(\delta, q; \boldsymbol{s})}_{\text{rank share}} \underbrace{G_n(\xi)}_{\text{firm gains}}$$

- Ordered shares: $0 < \gamma_n \leq \cdots < \gamma_2 < \gamma_1$.
- Comparative statics: $\uparrow \delta$ *steepens*, $\uparrow q$ *flattens* $\{\gamma_k\}$.
- *State separation:* all ξ–dependence sits in $(\bar{w}(\xi),\ G_n(\xi))$; $\{\gamma_k\}$ depend only on $(\delta, q; \boldsymbol{s})$.
- *Rank-difference identity:* for any $i < j$, $w_i - w_j = (\gamma_i - \gamma_j)\, G_n$.

Identification-ready implications

- Firm shocks: $\frac{\partial w_k}{\partial \xi} = \bar{w}_\xi + \gamma_k\, G_{n,\xi} \Rightarrow$ cross-rank slope in $\xi \propto \gamma_k$.
- Common vs. differential: level shifts $(\bar{w})$ are common; only G_n loads differentially via γ_k.
- Rank reshuffles (test): moving from k to $k' > k \Rightarrow$ weakly lower share.

No-delay logic

Earlier positions have stronger credible rejection (higher continuation without them) $\Rightarrow$ accept at rank-specific γ_k with strict ordering.

Core result: ordered shares

$$\gamma_1 > \gamma_2 > \cdots > \gamma_n, \qquad \text{whenever } \delta < 1 \text{ or } q > 0.$$

- *State-free order.* Ranking depends on $(\delta, q; \boldsymbol{s})$, not on ξ.
- *Wage ladder.* If $G_n(\xi) > 0$, then $w_k(\xi) = \bar{w}(\xi) + \gamma_k\, G_n(\xi)$ is strictly increasing in seniority.
- *Comparative statics.* $\uparrow \delta$ steepens the ladder; $\uparrow q$ flattens it.
- *Knife-edge equal shares.* Only at $\delta = 1$ and $q = 0$ do γ_k coincide.

Empirical bite / falsification

- Rank reshuffles: moving from k to $k' > k$ implies $\Delta w = (\gamma_{k'} - \gamma_k)\, G_n(\xi) < 0$ when $G_n(\xi) > 0$.
- Common shocks: cross-rank slope in ξ is proportional to γ_k.
- Equal-share benchmarks violate both signatures above.

Scope

Order holds for any fixed queue $\boldsymbol{s}$; changes in (δ, q) shift levels but *preserve* the ranking.

Ordered shares (proof intuition)

$$\gamma_1 > \gamma_2 > \cdots > \gamma_n \qquad \text{whenever } \delta < 1 \text{ or } q > 0.$$

- *Backward induction on m.* In the m-worker subgame, the no-delay offer at session m makes that worker just indifferent between accepting now and letting the game restart with $m-1$ workers.
- *Credible rejection is stronger earlier.* A rejection at an earlier position shrinks the workforce sooner and triggers more discount/breakdown ($\delta < 1$ or $q > 0$), so the firm's *restart loss* is larger earlier:

$$\Delta_k^F \equiv (\text{firm continuation if accept}) - (\text{if restart at } k) \quad \Rightarrow \quad \Delta_1^F > \Delta_2^F > \cdots > \Delta_n^F \geq 0.$$

- *No-delay pins the concession.* The minimal concession that deters rejection at session k is *increasing* in Δ_k^F. In particular, the no-delay offer gives the worker at least the firm's avoided loss:

$$\gamma_k \, G_k(\xi) \;\geq\; \Delta_k^F,$$

  hence γ_k is strictly larger for earlier k, yielding $\gamma_1 > \gamma_2 > \cdots > \gamma_n$.
- *Knife-edge.* If $\delta = 1$ and $q = 0$, then $\Delta_k^F = 0$ for all k; the credible-rejection threat vanishes and shares coincide (equal-share benchmark).

Comparative statics in (δ, q)

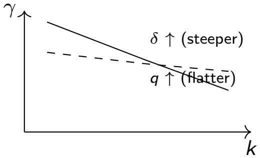
$$\frac{\partial \gamma_k}{\partial \delta} > 0, \qquad \frac{\partial \gamma_k}{\partial q} < 0.$$

$$\text{For } k < k' : \quad \frac{\partial}{\partial \delta}\left(\gamma_k - \gamma_{k'}\right) > 0, \qquad \frac{\partial}{\partial q}\left(\gamma_k - \gamma_{k'}\right) < 0.$$

- *Interpretation.* Higher patience ($\delta \uparrow$) steepens the ladder; more breakdown ($q \uparrow$) flattens it.
- *Rank sensitivity.* Effects are larger for early ranks (small k); gaps $\gamma_k - \gamma_{k+1}$ widen with δ and shrink with q.
- *State-free.* Depends on $(\delta, q; \boldsymbol{s})$, not on ξ.

One-graph intuition

Earlier ranks have stronger credible rejection; $\delta \uparrow$ amplifies it, $q \uparrow$ dampens it.



Empirical handle

Early–late gaps $\Delta(w_k - w_{k'}) = (\gamma_k - \gamma_{k'})G_n$ widen with δ, narrow with q.

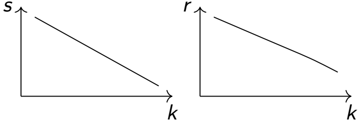
Mapping rank to empirical seniority

$$s_k = \frac{n-k}{n-1} \in [0,1], \qquad r_k = \ln \frac{n}{k}.$$

- *Strictly monotone.* With $k{=}1$ most senior, both s_k and r_k are strictly monotone in $k \Rightarrow$ all order & CS results in k carry to s or r.
- *Why two scales?* $s \in [0,1]$ pools across firm sizes; r (log) spreads top ranks—useful when upper-tail spacing matters.
- *Empirical recipe.* Compute k from within-firm tenure at (f,t); set $n{=}n_{ft}$. Ties $\rightarrow$ average ranks; if $n{=}1$, use $s{=}1$, $r{=}0$.
- *Link to wages.* Since $\gamma_k = \Gamma(s_k; \delta, q)$ is monotone in s, specs in s or r preserve ordered-share predictions for w_k.

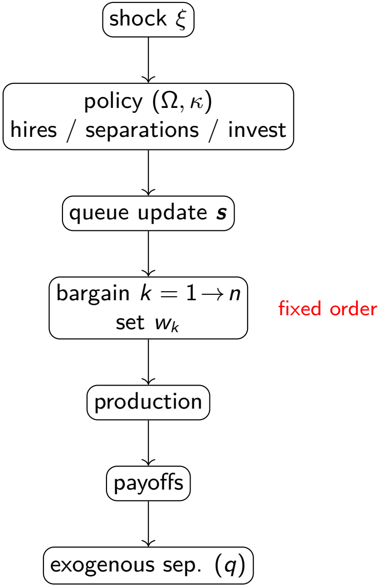
Takeaway

Same content, better comparability. Use s to compare slopes across firms; use r when top-rank spacing is key. Either way, ordered-shares implications are unchanged.



Timing within a period

1. **State & choices.** Shock ξ realizes. Manager chooses hires/separations Ω and specific investment κ.
2. **Bargaining.** Queue updates to $\boldsymbol{s}$; workers bargain in the fixed order $k = 1, \ldots, n$. No-delay equilibrium pins $\{w_k\}$.
3. **Production $\rightarrow$ payoffs $\rightarrow$ separations.** Output $F(n, \xi; \boldsymbol{s}, \kappa)$ is realized; wages are paid and the firm gets $\pi_n(\xi)$. Then exogenous separations occur (rate q), setting next period's n' and $\boldsymbol{s}'$.



Dynamic embedding: state and controls

State at start of period

$$x = (\xi,\ n,\ \boldsymbol{s},\ \boldsymbol{\kappa}), \qquad \boldsymbol{\kappa} = (\kappa_1, \ldots, \kappa_n).$$

Controls before bargaining
$\mathcal{S}$ (endogenous separations), h (hires), $\boldsymbol{I} = (I_1, \ldots, I_{n+h})$ (specific investment).

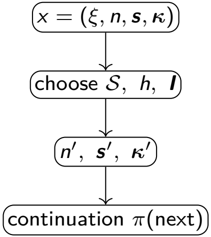
Laws of motion

$$\kappa_i' = (1-\rho)\kappa_i + I_i, \qquad \rho \in (0,1),$$
$$n' = n - |\mathcal{S}| + h.$$

Queue update: survivors shift up one rank; new hires enter at the bottom.

Why this matters for wages

Replacement costs, depreciation, and separations load *only* through the firm's continuation value $\pi_{m-1}(\xi)$. Hence ordered shares and their comparative statics carry over unchanged.



Firm problem (Bellman)

$$V^F(\xi, \boldsymbol{s}, \boldsymbol{\kappa}) = \max_{\mathcal{S},\, h,\, \boldsymbol{I} \geq 0} \mathbb{E}\Big[\underbrace{F(n^{\mathrm{pre}}, \xi; \boldsymbol{s}^{\mathrm{pre}}, \boldsymbol{\kappa}^{\mathrm{pre}})}_{\text{production}} - \underbrace{\textstyle\sum_{k \leq n^{\mathrm{pre}}} w_k}_{\text{wage bill}}$$

$$- \underbrace{\textstyle\sum_{i \leq n} I_i}_{\text{specific investment}} - \underbrace{\textstyle\sum_{i \in \mathcal{S}} c(\xi, s_i, \kappa_i)}_{\text{sep./replacement}} + \beta \underbrace{V^F(\xi', \boldsymbol{s}', \boldsymbol{\kappa}')}_{\text{continuation}} \Big].$$

Transitions / constraints: $\kappa_i' = (1-\rho)\kappa_i + I_i,\ \rho \in (0,1)$; $\quad n' = n - |\mathcal{S}| + h$; $\quad (\boldsymbol{s}^{\mathrm{pre}}, \boldsymbol{\kappa}^{\mathrm{pre}})$ from queue update.

Interpretation

- "pre" = after choosing $(\mathcal{S}, h, \boldsymbol{I})$ but *before* bargaining.
- In equilibrium: $w_k = \bar{w}(\xi) + \gamma_k(\delta, q; \boldsymbol{s})\, G_{n^{\mathrm{pre}}}(\xi)$, so the wage bill is affine in $G_{n^{\mathrm{pre}}}(\xi)$.
- Replacement costs, depreciation, and separations load only through π_{m-1} inside G_m; the order-driven ranking of shares is unchanged.

Wage block inside Bellman

$$\underbrace{G_{n^{\mathrm{pre}}}(\xi)}_{\text{new trade surplus}} = F\big(n^{\mathrm{pre}}, \xi; \boldsymbol{s}^{\mathrm{pre}}, \boldsymbol{\kappa}^{\mathrm{pre}}\big) - \pi_{n^{\mathrm{pre}}-1}(\xi) - n^{\mathrm{pre}}\,\bar{w}(\xi).$$

$$w_k(\xi) = \bar{w}(\xi) + \gamma_k\big(\delta, q; \boldsymbol{s}^{\mathrm{pre}}\big)\, G_{n^{\mathrm{pre}}}(\xi).$$

$$\sum_{k \le n^{\mathrm{pre}}} w_k = n^{\mathrm{pre}}\,\bar{w}(\xi) + \underbrace{\textstyle\sum_{k \le n^{\mathrm{pre}}} \gamma_k(\delta, q; \boldsymbol{s}^{\mathrm{pre}})}_{\Gamma_{n^{\mathrm{pre}}}}\, G_{n^{\mathrm{pre}}}(\xi).$$

- $\gamma_k \in (0,1)$ is pinned by order/queue, not by the state $\xi \Rightarrow$ the ranking comes from γ_k.
- Replacement costs, depreciation, and separations load *only* via $\pi_{n^{\mathrm{pre}}-1}$ inside G_m; ordered-shares and their comparative statics are unchanged.

Worker problem

$$v_k^W(\xi, s_k, \kappa_k) = \underbrace{w_k(\xi)}_{\text{current pay}} + \beta\,\mathbb{E}\Big[(1-\sigma)\underbrace{v_k^W(\xi', s_k', \kappa_k')}_{\text{stay at firm}} + \sigma\underbrace{\bar{v}(\xi')}_{\text{separate}}\Big].$$

- *Transitions.* $\kappa_k' = (1-\rho)\kappa_k + I_k$; s_k' from the queue update (survivors shift up; new hires enter at the bottom).
- *Separation.* $\sigma \in (0,1)$ is the end-of-period exogenous hazard; endogenous separations ($\mathcal{S}$) are chosen before bargaining and move the worker directly to $\bar{v}$.
- *Link to wages.* Using $w_k(\xi) = \bar{w}(\xi) + \gamma_k(\delta, q; \boldsymbol{s})\, G_n(\xi)$, seniority raises current pay and continuation (via s_k'), generating the tenure gradient we test.

Weak LIFO (exchange argument)

Claim (Weak LIFO). If separation/replacement cost $c(\xi, s, \kappa)$ is weakly increasing in seniority s and the production/continuation block is *symmetric in labels* (depends only on the number of survivors m and on the *multiset* of κ's, not on who holds which rank), then there exists an optimal policy that removes the least–senior workers first. If c is *strictly* increasing in s, optimal separations are LIFO a.s. (ties aside).

Why symmetry fits our environment. After choosing $|\mathcal{S}|$, the queue updates and surviving ranks are $\{1, \ldots, m\}$ deterministically, so the wage block is $\sum_{k \leq m} w_k \;=\; m\,\bar{w}(\xi) + \Gamma_m(\delta, q)\, G_m(\xi)$ with the slope Γ_m depending on m but *not* on which labels survive.

Exchange step. Consider any feasible policy that separates a more-senior i but keeps a less-senior $j > i$. Construct a policy that instead keeps i and separates j:

- *Current cost:* $c(\xi, s_i, \kappa_i) \geq c(\xi, s_j, \kappa_j)$ by c nondecreasing in $s \Rightarrow$ total separation cost weakly *falls*.
- *Production/continuation:* by symmetry in labels and the deterministic rank set $\{1, \ldots, m\}$ after the queue update, the production term and continuation value are *unchanged*.
- *Hence* the modified policy weakly improves the Bellman objective.

Iterating exchanges yields a LIFO policy with (weakly) minimal cost among all policies with the same m. If c is strictly increasing in s, every violation of LIFO is strictly dominated, so LIFO is optimal a.s.

Weak LIFO (proof steps)

Assumptions. $c(\xi, s, \kappa)$ weakly increasing in s; F symmetric in labels; the firm's value V^F has increasing differences in $(\text{keep}_i, -s_i)$ (supermodularity delivered by the SZ block and queue order).

Exchange step. Suppose an optimal policy separates i (senior, $s_i > s_j$) and keeps j (junior). Consider the swapped policy (separate j, keep i):

$$\Delta \text{ current} = \big[-c(\xi, s_j, \kappa_j)\big] - \big[-c(\xi, s_i, \kappa_i)\big] = c(\xi, s_i, \kappa_i) - c(\xi, s_j, \kappa_j) \;\geq\; 0,$$

$$\Delta \text{ continuation} \;\geq\; 0 \quad \text{by increasing differences (keeping the more senior weakly raises } V^F\text{)}.$$

So the swap weakly improves value—contradicting optimality. Hence some optimum never separates a senior while keeping a junior.

Monotone threshold. The problem forms a lattice and the objective is supermodular in $(\text{keep}, -s)$. By Topkis, there exists a threshold s^* with

$$\text{keep } i \iff s_i \geq s^*,$$

i.e. (weak) LIFO. If $c_s > 0$ (and strict single-crossing), the inequality is strict a.s. $\Rightarrow$ LIFO.

Existence of stationary MPE

Environment. State $x = (\xi, n, \boldsymbol{s}, \boldsymbol{\kappa})$; controls $a = (\mathcal{S}, h, \boldsymbol{I}) \in A(x)$. Wages come from the (no–delay) corrected SZ split: $w_k(\xi) = \bar{w}(\xi) + \gamma_k(\delta, q; \boldsymbol{s}^{\mathrm{pre}})\, G_{n^{\mathrm{pre}}}(\xi)$.

Bellman operator.

$$(\mathcal{T}V)(x) = \sup_{a \in A(x)} \Big\{ R(x,a) + \beta\, \mathbb{E}\big[V(x') \mid x, a \big] \Big\},$$

with stage payoff

$$R(x,a) = F(n^{\mathrm{pre}}, \xi; \boldsymbol{s}^{\mathrm{pre}}, \boldsymbol{\kappa}^{\mathrm{pre}}) - \sum_{k \le n^{\mathrm{pre}}} w_k - \sum_{i \le n} I_i - \sum_{i \in \mathcal{S}} c(\xi, s_i, \kappa_i).$$

Assumptions.

(A1) $\beta \in (0,1)$; $F, \bar{w}, c$ are bounded and continuous in their arguments.

(A2) $A(x)$ is nonempty, compact, and upper hemicontinuous in x (Berge).

(A3) SZ split is *single-valued and continuous* in $(\xi, n, \boldsymbol{s})$, hence R is continuous (affine in $G_{n^{\mathrm{pre}}}(\xi)$).

(A4) The transition kernel $P(\cdot \mid x, a)$ induced by queue update, ξ' and $\boldsymbol{\kappa}'$ is Feller (continuous in (x, a)).

Theorem (Existence and uniqueness of V^*; stationary MPE). Under (A1)–(A4), $\mathcal{T} : (B(X), \|\cdot\|_\infty) \to (B(X), \|\cdot\|_\infty)$ is a β–contraction (Blackwell's conditions: monotonicity and discounting). Hence there is a unique fixed point V^*, and by the measurable maximum theorem there exists a measurable selector $a^*(x) \in \arg\max\{\cdots\}$. The stationary profile $\big(a^*(\cdot),\, w(\cdot)\big)$ together with P constitutes a stationary Markov-perfect equilibrium.

Efficiency wedge (planner vs. decentralized)

Objects. $\Gamma_n = \sum_{k \le n} \gamma_k(\delta, q; \boldsymbol{s}), \qquad G_n(\xi) = F(n, \xi; \boldsymbol{s}, \boldsymbol{\kappa}) - \pi_{n-1}(\xi) - n\,\bar{w}(\xi).$

Wage bill: $\sum_{k \le n} w_k = n\,\bar{w}(\xi) + \Gamma_n\, G_n(\xi).$

Wedge (marginal value of G_n). planner $= 1$, firm $= 1 - \Gamma_n$.

FOC for any control a moving G_n: $\dfrac{\partial G_n}{\partial a} = \mathrm{MC}_a$ vs. $(1 - \Gamma_n)\dfrac{\partial G_n}{\partial a} = \mathrm{MC}_a$.

Implications

- If $0 < \Gamma_n < 1$: under-investment in retention/hiring/specific I; over-separation.
- Rank dependence: larger γ_k at top $\Rightarrow$ private choices tilt away from seniors when $c(\xi, s, \kappa)$ rises in s (weak LIFO link).
- Comparative statics: $\Gamma_n \uparrow$ with δ, $\Gamma_n \downarrow$ with $q \Rightarrow$ wedge $1 - \Gamma_n$ shrinks with δ, grows with q.

Sufficient-statistics predictions

$$\mathrm{PT}_k \equiv \frac{\partial w_k}{\partial \xi} = \bar{w}'(\xi) + \gamma_k(\delta, q; \boldsymbol{s})\, G_n'(\xi).$$

- *Monotone pass-through.* If $G_n'(\xi) > 0$, then PT_k increases with seniority s_k. Differences are $(\gamma_{k'} - \gamma_k)\, G_n'(\xi)$.
- *Comparative statics.* The slope in s steepens with δ and flattens with q: $\partial(\mathrm{PT}_{k'} - \mathrm{PT}_k)/\partial\delta > 0$, $\partial(\cdot)/\partial q < 0$.
- *Rank shocks.* A switch $k \to k'$ yields a discrete jump $\Delta w = (\gamma_{k'} - \gamma_k)\, G_n(\xi)$ with no pre-trend when ξ is smooth and ranks move quasi-randomly.
- *Separations.* Optimal exit obeys (weak) LIFO: separation hazard $\downarrow$ in s; cutoffs $c^\star(\xi, s, \kappa)$ increase in s.

Falsification targets

Equal-share benchmarks imply a flat PT–s profile and zero rank-shock jumps; either violation rejects them in favor of ordered shares.

Empirics preview (Appendix): rank shocks $\Rightarrow$ jumps; PT steeper for seniors; equal-share benchmarks falsified.

Falsification: Rolodex/Shapley (equal share)

Null (equal share). $\gamma_k \equiv \frac{1}{n}$ for all k (Rolodex/Shapley/Nash). Hence $\mathrm{PT}_k = \bar{w}'(\xi) + \frac{1}{n} G_n'(\xi)$ is flat in s, and rank–shock jumps are $\Delta w = (\gamma_{k'} - \gamma_k) G_n(\xi) = 0$.

Test 1: Slope in s (sufficient statistic). Estimate $\mathrm{PT}_{kft} = \alpha_f + \alpha_t + \beta\, s_{kft} + u_{kft}$. Equal share predicts $\beta = 0$; our mechanism predicts $\beta > 0$ and steeper with δ, flatter with q.

Test 2: Rank–permutation placebo. Within firm–time, randomly permute $\{s_{kft}\}$ and re-estimate β. Ordered shares $\Rightarrow$ $\widehat{\beta}$ collapses toward 0; equal share $\Rightarrow$ no change (already 0).

Test 3: Rank-shock event study (corollary). For quasi-random $k \to k'$, $\Delta w_{kft} = (\gamma_{k'} - \gamma_k) G_n(\xi_t)$ at $t = 0$ with *no pre-trends*; equal share $\Rightarrow$ $\Delta w = 0$.

Alternative protocols (rejections)

- **Simultaneous Nash (all workers + firm).** One–shot split of total surplus $\Rightarrow \gamma_k \equiv \frac{1}{n}$. Predictions: PT_k flat in s; rank–shock jumps $\Delta w = 0$. Rejected by positive PT–s slope and observed jumps.
- **Rolodex/Shapley (orderless).** Orderless equal division in expectation $\Rightarrow \gamma_k$ identical. Same predictions: PT flat in s, no rank–shock jumps. Rejected by PT–s gradient and permutation placebo (slope vanishes under random ranks).
- **Union common contract.** Single wage rule $w^U(\cdot)$ within contract cells (skill/tenure bands) $\Rightarrow$ dispersion compressed: *within a cell* PT_k is flat in s; changes only at cell switches/renegotiations. Empirical check: within-cell PT–s slope > 0 and rank–shock jumps at fixed cells $\Rightarrow$ rejects binding common-contract.

Comparative-statics levers (mapping to δ and q)

$$\frac{\partial \gamma_k}{\partial \delta} > 0, \qquad \frac{\partial \gamma_k}{\partial q} < 0 \quad \Rightarrow \quad \text{seniority slope in } s \text{ steepens with } \delta \text{ and flattens with } q.$$

Levers that plausibly shift δ (relational horizon) or q (breakdown risk):

- **Training intensity / tenure ladders** $\Rightarrow$ longer horizon $\Rightarrow \delta \uparrow \Rightarrow$ steeper ordered shares, larger senior pass-through.
- **Vacancy fill times (replacement speed).** Slow fills make impasse costly $\Rightarrow q \downarrow$ (continue rather than break) $\Rightarrow$ steeper seniority premia.
- **Dispute resolution (mediation/arbitration, HR/works councils).** Fewer impasses $\Rightarrow q \downarrow \Rightarrow$ steeper slopes and larger rank-shock jumps.
- **Union coverage (institutional channel).** Formal grievance/arbitration $\Rightarrow q \downarrow$ (steepens); common-contract *compression* within cells (test: within-cell PT–s slope).
- **Employment protection (EPL).** Harder to sever $\Rightarrow q \downarrow$ and longer expected match $\Rightarrow \delta \uparrow \Rightarrow$ steeper seniority gradient.

Design. Interact seniority s with each lever (or bin by lever) and test: slope in s rises where δ is higher / q lower; equal-share benchmarks predict no changes.

Conclusion (theory-first)

Mechanism. Corrected SZ with a fixed queue $\Rightarrow$ order-dependent shares; no-delay equilibrium well defined.
Ordered shares. $\gamma_1 > \gamma_2 > \cdots > \gamma_n$ (state-free). Wages factor: $w_k(\xi) = \bar{w}(\xi) + \gamma_k\, G_n(\xi)$ with all dynamics packed into π_{n-1} via G_n.
Dynamics. Weak LIFO from an exchange argument; monotone separation thresholds. $\uparrow \delta$ steepens, $\uparrow q$ flattens the seniority ladder.
Sufficient statistics & ID. Map rank $\rightarrow$ seniority s; pass-through $\uparrow$ in s; rank shocks $\Rightarrow$ discrete wage steps, no pre-trends.
Efficiency. Planner–decentralized wedge $1 - \Gamma_n \Rightarrow$ under-provision of retention/investment, over-separation (ties to LIFO).

One-line takeaway

Order + one surplus term G_n organize both the theory and the measurement; the model yields sharp, falsifiable gradients in seniority.

Extension E.1: Risk-averse workers

Setup. Worker k has vNM utility $u(w)$ with $u' > 0 > u''$; the firm is risk-neutral.

Bilateral split (Rubinstein–BRW in utility): Let $\varphi_u(\delta, q) \in (0,1)$ be the worker's bilateral *utility*-surplus share. Then $\varphi_u < \varphi$ (risk-neutral benchmark), $\frac{\partial \varphi_u}{\partial \delta} > 0$, and $\frac{\partial \varphi_u}{\partial q} < 0$.

$$\underbrace{w_k(\xi)}_{\text{cash}} = \bar{w}(\xi) + \underbrace{\gamma_k^u(\delta, q; \boldsymbol{s})}_{\text{compressed shares}} \; G_n(\xi), \qquad 0 < \gamma_n^u < \cdots < \gamma_1^u < 1, \;\; \gamma_k^u < \gamma_k.$$

- *Order preserved.* Earlier positions still capture strictly larger shares whenever $G_n(\xi) > 0$: $\gamma_1^u > \gamma_2^u > \cdots > \gamma_n^u$.
- *Comparative statics.* $\frac{\partial \gamma_k^u}{\partial \delta} > 0$, $\frac{\partial \gamma_k^u}{\partial q} < 0$, with larger semi-elasticities for smaller k. Risk aversion compresses the seniority ladder relative to the risk-neutral case.
- *Empirical bite.* Pass-through gradient in seniority attenuates where wage risk is high. Recommended check: interact shock×seniority with a risk proxy (bonus/commission share, wage variance):

$$\Delta w_{ijt+\tau} \sim \Delta \ln Y_{jt} \cdot s_{ijt} \times \mathrm{Risk}_j \quad \Rightarrow \quad \text{coefficient } <0.$$

Extension E.2 — Firing costs & employment protection (EPL)

Modeling. Let $L > 0$ be a per-separation transfer from firm to worker.

Dynamic effect (value matching). In the baseline dynamic model, L raises workers' outside options and thus wages, *but* the optimal firing cutoff $p^\star$ is unchanged: the layoff-discouragement (direct) and wage-increase (indirect) effects exactly offset at the margin.

Translation to corrected SZ.

$$\boxed{w_k^{\mathrm{EPL}}(\xi) = \bar{w}(\xi; L) \; + \; \gamma_k(\delta, q; \mathbf{s}) \, G_n(\xi; L)}$$

- Level shift in wages at all ranks; $\bar{w}$ and G_n move with L.
- Shares unchanged: the *order and values* $\{\gamma_k\}$ are unchanged $\Rightarrow$ seniority gradient preserved.
- Employment dynamics: with the firing cutoff as the main margin, moderate EPL leaves separation thresholds essentially unchanged; pass-through gradients in s survive.

Empirical check (clean test).

- Interact demand shocks with seniority and EPL exposure: $\Delta \ln Y_{jt} \cdot s_{ijt} \cdot \mathrm{EPL}_j$.
- Prediction: level shifts in w at given s; *negligible* effects on estimated separation thresholds.

Extension E3: Union coverage & collective bargaining

Union logic (egalitarian pooling / two-tier). Coverage imposes a common wage for members and a distinct entry wage. In corrected-SZ terms this is a *compressed share vector*:

$$w_k^U(\xi) = \bar{w}(\xi) + \gamma_k^U\, G_n(\xi), \qquad \gamma_k^U = \begin{cases} \gamma_{\text{inc}}, & \text{incumbents } (k \le n^{\text{inc}}), \\ \gamma_{\text{ent}}, & \text{entrants } (k > n^{\text{inc}}), \end{cases} \qquad \gamma_{\text{inc}} > \gamma_{\text{ent}}, \ \ \{\gamma_k^U\} \text{ flatter than } \{\gamma_k\}.$$

Implications vs. non-union:

- Higher level, flatter slope. Wage bill rises; pass-through in s is *compressed* within incumbents; the big gap is incumbents vs entrants.
- Rank reshuffles. Within incumbents, rank swaps change little: $\Delta w \approx 0$; discrete jumps at the entry margin.
- Testable sign. In Δw_{ift}, the interaction $\Delta\xi_{ft} \times s_{ift} \times \text{Union}_{ft}$ is *negative* (flatter); an $\Delta\xi_{ft} \times \text{Union}_{ft}$ term is *positive* for incumbents.
- Order preserved. $\gamma_1^U \ge \cdots \ge \gamma_n^U$ (often two-tier); LIFO and (δ, q) comparative statics unaffected.

When could unions steepen? Under right-to-manage with very patient leadership that tilts shares to seniors; absent that, *political equalization* predicts flattening.

Extension E4: Teams and internal labor markets

Setup. Partition workers into teams $\tau = 1, \dots, T$ with sizes n_τ, within-team ranks $k \in \{1, \dots, n_\tau\}$, and team knowledge κ^τ. Let production decompose into a firmwide part plus team parts:

$$F(n, \xi; \boldsymbol{s}, \boldsymbol{\kappa}) = F^{\text{firm}}(\xi) \; + \; \sum_{\tau=1}^{T} F_\tau(n_\tau, \xi_\tau; \boldsymbol{s}^\tau, \kappa^\tau),$$

so the gains-from-trade split as $G_n(\xi) = G^{\text{firm}}(\xi) + \sum_\tau G_{n_\tau,\tau}(\xi_\tau)$.

Wages. Corrected-SZ within each team:

$$w_{k\in\tau}(\xi) = \bar{w}(\xi) + \underbrace{\gamma_{k,\tau}(\delta, q; s^\tau)\, G_{n_\tau,\tau}(\xi_\tau)}_{\text{team-specific pass-through}} + \underbrace{\phi_\tau \; G^{\text{firm}}(\xi)}_{\text{common component}} \quad , \quad \gamma_{1,\tau} > \cdots > \gamma_{n_\tau,\tau}.$$

Predictions.

- Within-team ordering: for each τ, the wage ladder is strictly increasing in within-team seniority k whenever $G_{n_\tau,\tau} > 0$.
- Team shocks vs. firm shocks: a shock to ξ_τ steepens the gradient only in team τ (via $G_{n_\tau,\tau}$); a firmwide shock moves all teams in parallel (via G^{firm}).
- Heterogeneous slopes: teams with more complementarity/retention value (curvature of F_τ) display steeper $\{\gamma_{k,\tau}\}$.
- Separations: if $c_s \geq 0$ within teams, weak LIFO applies *teamwise*; new hires enter at the bottom of team's queue.

Empirical mapping. Use product-line / project shocks as ξ_τ; estimate $\Delta w_{if\tau t}$ on $\Delta\xi_{\tau t} \times s_{if\tau t}$ with firm-time FE to net out G^{firm}.

Extension E5: Abundant trained replacements

Idea. If an exiting worker can be *immediately* replaced by a trained hire (before production) at effective cost c_R with probability $\phi \in [0,1]$, the incremental surplus from keeping the marginal incumbent shrinks.

Single object that moves. For the m–worker subgame,

$$G_m^{(\phi)}(\xi) = \underbrace{F(m,\xi;s,\kappa) - \pi_{m-1}(\xi) - m\,\bar{w}(\xi)}_{G_m(\xi)} \longrightarrow G_m^{(\phi)}(\xi) \approx (1-\phi)\,G_m(\xi) - \phi\,c_R.$$

Invariance. Order shares $\gamma_k(\delta,q;s)$ are unchanged (fixed by extensive form); only the common factor G_n is rescaled.

Wages.

$$w_k(\xi) = \bar{w}(\xi) + \gamma_k(\delta,q;s)\,G_n^{(\phi)}(\xi), \qquad \gamma_1 > \cdots > \gamma_n.$$

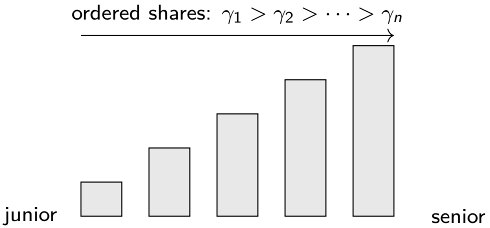
As $\phi\uparrow$ and $c_R\downarrow$, pass-through and the seniority gradient flatten (via $G_n^{(\phi)}\downarrow$), while the strict ordering of shares survives.

Separations (weak LIFO). If replacement weakens c_s (effective replacement cost rises less in s), LIFO *weakens* but the threshold monotonicity logic still applies; with $c_R\approx 0$ and $\phi\approx 1$, separation cutoffs converge across ranks.

Empirical fingerprints.

- Flatter wage–seniority slopes and lower rent pass-through in occupations/locations with deep trained pools (temp agencies, modular tasks, standardized credentials).
- Weaker retention of seniors after positive shocks; faster post-shock headcount recovery via hires.
- Design: interact firm shocks with seniority and a proxy for replacement depth (local stock of certified workers, pipeline capacity, temp-agency penetration).

THANK YOU!



Feedback welcome (anytime, I'll be around for a while).

APPENDIX: From model objects to estimands

Model link. $w_k(\xi) = \bar{w}(\xi) + \gamma_k(\delta, q; s_k)\, G_n(\xi) \Rightarrow$

$$\frac{\partial w_k}{\partial \ln Y} = \underbrace{\gamma_k(\delta, q; s_k)}_{\text{order/queue}} \underbrace{\psi}_{\psi \equiv \partial G_n / \partial \ln Y} .$$

A. Sufficient-statistics (local projections)

$$\Delta w_{ift+\tau} = \beta_{0,\tau}\, \Delta \ln Y_{ft} + \beta_{1,\tau} \left(\Delta \ln Y_{ft} \times s_{ift} \right) + \text{FE} + X'_{ift}\theta + \varepsilon_{ift+\tau}.$$

Mapping (e.g., $\gamma(s) = \Gamma_0 + \Gamma_1 s$*):* $\beta_{0,\tau} \approx \Gamma_0\, \psi_\tau, \quad \beta_{1,\tau} \approx \Gamma_1\, \psi_\tau.$

B. Plug-in G_n (accounts/queue stats)

Compute $\widehat{G}_{ft}$ from firm data, then

$$\Delta w_{ift+\tau} = \lambda_\tau \left(\widehat{G}_{ft} \times s_{ift} \right) + \mu_\tau\, \widehat{G}_{ft} + \text{FE} + X'_{ift}\theta + u_{ift+\tau}.$$

Mapping: λ_τ traces the slope of $\gamma(s)$; μ_τ the level.

Identification notes. Within-firm variation in s; exogenous $\Delta \ln Y_{ft}$ (or IV); rich FE (worker, firm×t, rank-band×t) absorb $\bar{w}(\xi)$ and composition; event-study τ to show no pretrends; cluster at firm.

APPENDIX: Baseline specification

Baseline (compact, equivalent re-parameterization).

$$\Delta w_{ijt+\tau} = \underbrace{\beta_L \, \Delta \ln Y_{jt}}_{\text{pass-through at } s=0} + \underbrace{\beta_\Delta \left(\Delta \ln Y_{jt} \times s_{ijt}\right)}_{\text{slope in seniority}} + \lambda \left(\widehat{G}_{jt} \times s_{ijt}\right) + \phi_1 \, w_{ijt} + \phi_2 \, \overline{w}^{\,\text{stable}}_{-i,j,t} + \Gamma' X_{ijt} + \mu_i + \mu_j + \mu_t + \varepsilon_{ijt+\tau}.$$

Equivalence to two-interaction form: using $\Delta \ln Y_{jt} \times s_{ijt}$ and $\Delta \ln Y_{jt} \times (1 - s_{ijt})$, report $\beta_L \equiv \beta_2$ and $\beta_\Delta \equiv \beta_1 - \beta_2$.

Interpretation & identification

- β_L = pass-through to least senior ($s=0$); β_Δ = increase in pass-through per unit s ($\Rightarrow$ ordered-shares slope).
- λ traces $\gamma(\cdot)$ via plug-in $\widehat{G}_{jt}$ (accounts/queue stats).
- Controls: w_{ijt} absorbs mean reversion; $\overline{w}^{\,\text{stable}}_{-i,j,t}$ for peer/firm wage drift; X_{ijt} for composition.
- Fixed effects: worker (μ_i), firm (μ_j), time (μ_t). Robustness: firm×t or rank-band×t FEs.
- Inference: cluster at firm (or firm×sector); event-study leads/lags to show no pre-trends.

APPENDIX: IV & first stage (shift–share)

Instrument (Bartik / shift–share).

$$Z_{jt} \;=\; \sum_k \omega_{jk,t_0}\,\Delta g_{kt}, \qquad \sum_k \omega_{jk,t_0} = 1, \;\; t_0 \text{ pre-dates } t.$$

First stage.

$$\Delta \ln Y_{jt} \;=\; \pi\, Z_{jt} \;+\; \underbrace{\sum_k \omega_{jk,t_0}\,\mu_{kt}}_{\text{exposure-saturated sector}\times\text{time FE}} \;+\; \Psi' W_{jt} \;+\; \mu_j \;+\; \mu_t \;+\; \upsilon_{jt}.$$

Construction & identification

- *Weights:* baseline exposures ω_{jk,t_0} (frozen at t_0 to avoid endogenous drift).
- *Shocks:* Δg_{kt} are sectoral shocks plausibly orthogonal to firm j's idiosyncratic choices.
- *Leave-one-out:* compute $\Delta g_{k,-j,t}$ excluding j's contribution and set $Z_{jt}^{\mathrm{LOO}} = \sum_k \omega_{jk,t_0} \Delta g_{k,-j,t}$.
- *Exposure saturation:* include $\sum_k \omega_{jk,t_0} \mu_{kt}$ (or industry$\times$time FE) to soak up common sectoral movements.
- *Exogeneity:* $\mathbb{E}[\Delta g_{kt} \mid \mu_{kt}, W_{jt}, \mu_j, \mu_t] = 0$ and ω_{jk,t_0} predetermined.

Diagnostics & inference

- Report first-stage F (or Kleibergen–Paap rk F); show $\hat{\pi} \neq 0$.
- Use weak-IV–robust CIs in second stage (Anderson–Rubin and CLR).
- Cluster at firm (and/or sector) level; allow serial correlation within j.

APPENDIX: Identification (design-based)

Design assumptions.

D1 *Conditional exogeneity of shocks.*

$$\mathbb{E}[\varepsilon_{ijt+\tau} \mid \Delta \ln Y_{jt}, W_{jt}, X_{ijt}, \mu_i, \mu_j, \mu_t] = \mathbb{E}[\varepsilon_{ijt+\tau} \mid W_{jt}, X_{ijt}, \mu_i, \mu_j, \mu_t].$$

D2 *Relevance.* With FE and rich W_{jt}, $\mathrm{Cov}(Z_{jt}, \Delta \ln Y_{jt}) \neq 0$ (strong first stage).

D3 *Exclusion.* Z_{jt} affects outcomes only through $\Delta \ln Y_{jt}$: $\mathrm{Cov}(Z_{jt}, u_{ijt+\tau}) = 0$ given exposure-saturated sector$\times$time FE and controls.

D4 *Within-firm–time rank orthogonality.* s_{ijt} is a permutation index constructed from pre-t tenure; within (j, t), $\mathrm{Cov}(s_{ijt}, \varepsilon_{ijt+\tau} \mid X_{ijt}, \mu_j, \mu_t) = 0$.

How we make D1–D4 credible

- **Predetermination:** fix ω_{jk,t_0} at $t_0 < t$; build s_{ijt} from tenure at $t-1$.
- **Exposure saturation:** include $\sum_k \omega_{jk,t_0} \mu_{kt}$ (sector$\times$time FE) and firm FE μ_j.
- **Leave-one-out IV:** use Z_{jt}^{LOO} to cut mechanical feedback from firm j.
- **Placebos & leads:** stacked-lead event studies ($\ell > 0$) $\Rightarrow$ no pre-trends; rank *permutation* tests (shuffle s_{ijt} within (j, t)) $\Rightarrow$ null effects.
- **Diagnostics:** report Kleibergen–Paap rk F; weak-IV–robust AR/CLR CIs; cluster at firm (and sector) level.

APPENDIX: Event studies for rank shocks

Event definition (within firm j). A *rank shock* at time t is any within-j reshuffle with $\Delta s_{ijt} \neq 0$ for worker i (e.g., hires/separations/promotions). Keep first qualifying event per (i, j); trim overlapping windows; require $|\Delta s_{ijt}| \geq \tau_s$.

Stacked event-window specification. Normalize $\tau = -1$.

$$\Delta w_{ijt+\tau} = \sum_{\ell=-L}^{-2} \alpha_\ell \, \mathbf{1}\{\tau = \ell\} \cdot \Delta s_{ijt} + \sum_{\ell=0}^{H} \alpha_\ell \, \mathbf{1}\{\tau = \ell\} \cdot \Delta s_{ijt} + \Gamma' X_{ijt} + \mu_i + \mu_j + \mu_t + u_{ijt+\tau}.$$

Variant (directional effects): replace Δs_{ijt} by $\mathbf{1}\{\Delta s_{ijt} > 0\}$ and $\mathbf{1}\{\Delta s_{ijt} < 0\}$ with coefficients $\alpha_\ell^{\text{up}}, \alpha_\ell^{\text{down}}$.

Construction & diagnostics

- X_{ijt}: pre-trend controls (baseline wage, tenure bins, age), firm $\times$ industry trends if desired.
- Cohort the events by calendar time; stack windows; cluster SEs at firm (and worker) levels.
- Placebos: lead coefficients α_ℓ for $\ell < 0 \approx 0$ (no pre-trends).

Model-mapped predictions

- Discrete jump at $\tau = 0$: $\alpha_0 \approx (\gamma_{k'} - \gamma_k)\, G_n(\xi)$ when rank moves $k \to k'$.
- No pre-trends: $\alpha_\ell \approx 0$ for all $\ell < 0$.
- Persistence: post-τ path flat if $G_n(\xi)$ stable; amplifies in high-δ/low-q subsamples.

APPENDIX: Local projections & SIMEX

Local projections by seniority quantiles

$$\Delta w_{ijt+\tau} = \theta_{\tau,q}\, \Delta \ln Y_{jt} + \Gamma' X_{ijt} + \mu_i + \mu_j + \mu_t + u_{ijt+\tau}, \qquad q \in \{0.1, 0.5, 0.9\}.$$

- Quantiles q formed on s_{ijt} (within-firm or pooled, fixed over window).
- Cluster SEs at firm (and worker); horizons $\tau = 0, 1, \ldots, H$ stacked.
- Predictions: $\theta_{\tau,0.9} > \theta_{\tau,0.1}$ for $\tau \geq 0$; asymmetry under negative shocks (smaller magnitudes).

Measurement error in s and SIMEX for the first stage

Adjacent-swap misclassification rate p attenuates seniority slopes:

$$\beta_s^{\text{OLS}} \approx \beta_s(1 - 2p).$$

SIMEX (on shifter/first stage):

1. Inject calibrated noise: $Z_{jt}^{(\lambda)} = Z_{jt} + \sqrt{\lambda}\, \varepsilon_{jt}$ for $\lambda \in \Lambda$.
2. Re-estimate first stage $\Delta \ln Y_{jt}$ on $Z_{jt}^{(\lambda)}$ (same W_{jt}, FEs); carry fitted values/IV into the LP to obtain $\theta_{\tau,q}(\lambda)$.
3. Fit a low-order polynomial in λ and extrapolate to $\lambda \to -1$ (zero noise) to recover $\theta_{\tau,q}^{\text{SIMEX}}$.

Notes: leave-one-out exposure for Z_{jt}; weak-IV robust AR/CLR at each τ; bootstrap across firms for SIMEX bands.

APPENDIX: Industry primitives (FPT) & augmented wage model

Recovering primitives from job durations (first passage time)

Completed spell lengths in industry g identify two primitives:

$$\Omega_g \text{ (initial surplus distance)}, \qquad \pi_g \text{ (drift of fundamentals)}.$$

Estimate (Ω_g, π_g) by fitting an FPT family to completed jobs (exposure-saturated, leave-one-out if needed). There is a monotone mapping from (Ω_g, π_g) to (i) specific-knowledge stocks and (ii) trend in fundamentals, so these serve as sufficient summary shifters for how shocks pass through with seniority.

Augmenting the wage equation with primitives (interaction tests)

Let T_{ijt} be tenure. We estimate

$$\Delta w_{ijt+\tau} = \beta_L \, \Delta \ln Y_{jt} + \beta_\Delta \, \Delta \ln Y_{jt} \cdot s_{ijt} + \theta_1 \, s_{ijt} \cdot \Omega_{\mathrm{ind}(j)} + \theta_2 \, s_{ijt} \cdot \pi_{\mathrm{ind}(j)} \cdot T_{ijt}$$

$$+ \theta_3 \, \pi_{\mathrm{ind}(j)} \cdot T_{ijt} + \lambda \, \widehat{G}_{jt} \cdot s_{ijt} + \mathrm{FE} + \mathrm{controls} + \varepsilon_{ijt+\tau}.$$

Explain–away test: if (Ω, π) capture persistent knowledge and drift, then

$$\widehat{\beta}_\Delta \text{ shrinks in magnitude once } (\Omega, \pi) \text{ enter.}$$

Interpretation: $\theta_1, \theta_2 > 0 \;\Rightarrow$ steeper seniority pass-through in industries with larger surplus distance and faster drift; θ_3 captures level tilts with tenure even absent seniority interaction.

Housekeeping: firm×year and worker FE; two-way clustering or firm-level wild bootstrap; sample-splitting for generated (Ω, π); delete-one-industry jackknife; pre-trend checks in industries with stable (Ω, π).